

## Pascal trihedron and Collatz algorithm.

### Hubert Schaetz

**Abstract** Congruencies are at the centre of the mechanism of descent below the initial flight altitude for the Collatz algorithm. The number of elements of classes of constant altitude flight time is assessed through a routine in the form of a ‘Pascal trihedron’.

#### Trièdre de Pascal et algorithme de Collatz.

**Résumé** Les congruences sont au cœur du mécanisme de descente sous l’altitude initiale dans l’algorithme de Collatz. L’effectif des classes à temps de vol en altitude constant s’évalue grâce à une routine en forme de «trièdre de Pascal».

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#### 1 1 Formulation of the conjecture and framework.

The Collatz conjecture is also called Syracuse conjecture, conjecture of Ulam, Czech conjecture or 3x+1 problem. The series  $u_n$  of an integer N is defined by recurrence, as follows [2] :

$$\begin{aligned} u_0 &= N \\ u_{n+1} &= u_n/2 \quad \text{if } u_n \text{ is even} \\ u_{n+1} &= 3u_n + 1 \quad \text{if } u_n \text{ is odd} \end{aligned}$$

The conjecture states that, for every N, there is a finite index n such as :

$$u_n = 1$$

One can then define :

- the time or duration of flight (TF) : it is the smallest index n such that  $u_n = 1$ .
- the time of flight in altitude (TFA) : it is the smallest n such that  $u_n < u_0$ .
- the number (v) of odd steps (I) of the altitude flight time : this is the number of multiplications by 3 (plus 1) before  $u_n < u_0$ .
- the number (w) of even steps (P) of the altitude flight time : this is the number of divisions by 2 before  $u_n < u_0$ .

We have of course ;

$$\text{TFA} = v+w = n$$

For N = 3 for example, the sequence will be the following :

3	10	5	16	8	4	2
I	P	I	P	P	P	

2 is the first instance smaller than 3 in the sequence 3, 10, 5, 16, 8, 4, 2.

The sequence has v = 2 odd steps I and w = 4 even steps P and the flight time in attitude is here 6.

Riho Terras [3] demonstrated in 1976 the null density of Collatz exceptions. The present article contribution rest on a better comprehension of the mechanism of descent under the initial flight altitude and especially the discovery of the ‘‘Pascal trihedron’’ which governs the process.

No proof of any kind is undertaken here as it is the subject of another article [1].

#### 2 Mechanism of flight descent.

The flight time (TF) of a number N is finite if its flight time in attitude (TFA) is finite and if the time flight in attitude is finite for any number from 2 to N-1. If any time flight in attitude is finite, the Collatz conjecture is then true. The time flight (TF) concept is thus not always useful et it will not be used underneath.

The table of data at appendix 1 exhibits a number of redundancies in the altitude flight times. Thus 1, 3, 6, 8 and 11 appear

several times.

The explication is the following :

For  $N = 0 \bmod 2$ , we get applying the Collatz algorithm  $N \rightarrow N/2$  and  $N/2$  is inferior to  $N$ , hence an altitude flight time of 1. Any even number has this property.

For  $N = 1 \bmod 4$ , we get  $1+2^k \rightarrow 4+3.2^k \rightarrow 2+3.2^k \rightarrow 1+3k$  and  $1+3k$  is inferior to  $1+2^k$  (which is not the case before that), hence an altitude flight time of 3. Any number  $1 \bmod 4$  has thus this property.

In this logic, it is relatively easy to find for a number chosen in advance, the kind of operations which brings him back below its initial value and then associate to it a family modulo a power of 2. It is not the same of course for the generalization of the concept (which is equivalent to the Collatz conjecture).

The appendix 2 gives some of these families.

We propose here to enumerate the staffs of these families, the members of same family being called associates.

### Proposition 1

Let us have  $N$  a positive integer with finite time flight. Let us have  $v$  and  $w$  the number of odd and even steps of the Collatz algorithm for  $N$ . If  $N$  is less than  $2^w$ , then the altitude time flight of  $N+k.2^w$  is that of  $N$  for any positive integer  $k$  with the same numbers of even and odd steps.

### Proposition 2

Let us have  $N$  any number with finite flight time composed of  $v$  odd steps and  $w$  even steps. The number  $w$  of even steps of flight in altitude is then linked to the number  $v$  of odd steps of flight in altitude by the relationship :

$$w = \text{int}((\ln(3)/\ln(2)).v)+1 \quad (1)$$

Note: The values of  $\Delta w$ , variation between two consecutive  $w$ , are equal to 1 or 2 exclusively (when  $v$  changes to  $v+1$ ).

### Recapitulative data table

Let us summarize in a table some initial values :

nb of odd steps of TFA = $v$	nb of even steps of TFA = $w = w(v)$	TFA	Modulo $2^w$ (mod)	Smallest $N$	Number of associates $\#(v)$
0	1	1	2	2	1
1	2	3	4	1	1
2	4	6	16	3	1
3	5	8	32	11	2
4	7	11	128	7	3
5	8	13	256	39	7
6	10	16	1024	287	12
7	12	19	4096	231	30
8	13	21	8192	191	85
9	15	24	32768	127	173
10	16	26	65536	359	476
11	18	29	262144	511	961
12	20	32	1048576	239	2652
13	21	34	2097152	159	8045
14	23	37	8388608	639	17637
15	24	39	16777216	283	51033
...	...	...	...	...	...

### **3 Infinite sum associated with the Collatz algorithm.**

The associates are necessarily all distinct. Their quantities and relative proportions are summed up below :

$$s = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^4} + \frac{2}{2^5} + \frac{3}{2^7} + \frac{7}{2^8} + \frac{12}{2^{10}} + \frac{30}{2^{12}} + \frac{85}{2^{13}} + \frac{173}{2^{15}} + \frac{476}{2^{16}} + \frac{961}{2^{18}} + \frac{2652}{2^{20}} + \frac{8045}{2^{21}} + \frac{17637}{2^{23}} + \dots$$

If the Collatz conjecture is true, the set of integers is necessarily described and the sum is 1 as shown previously. This is formally written using these article notations :

$$\sum_{v=0}^{+\infty} \frac{\#(v)}{2^w} = 1 \quad (2)$$

### Proposition 3

If the infinite sum  $\sum \#(v)/2^{w(v)}$  equals 1, the density of numbers N not satisfying the Collatz conjecture is null.

The converse is wrong as one can have a finite number of exceptions to the Collatz algorithm while keeping an infinite sum which tends to 1.

### **4 Enumeration of associates. The ultimate conquest.**

The title of the paragraph paraphrases another author. To find the families staffs  $\#(v)$  is indeed not trivial at the start of the investigation.

Let us again consider the sequences I and P earlier mentioned and IP type clusters since to any odd step succeeds always an even step. For a given v, we class the sequences as follows :

	Sequences
C <sub>1</sub>	IP, IP, IP, ..., IP, P, ..., P
C <sub>2</sub>	IP, IP, ..., IP, P, X, ..., X, P
...	...
C <sub>v-3</sub>	IP, IP, IP, P, X, ..., X, P
C <sub>v-2</sub>	IP, IP, P, X, ..., X, P
C <sub>v-1</sub>	IP, P, X, ..., X, P

Step X can as well be of type IP or of type P. Only matters us the number of steps IP without interruption at the beginning of routine. The cardinal of  $\#C_i(v)$  of each class is then accounted.

v	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
w	1	2	4	5	7	8	10	12	13	15	16	18	20	21	23	24
$\Delta w$	1	2	1	2	1	2	2	1	2	1	1	2	2	1	2	1
$\Delta w_{\text{prec1}}$		1	2	1	2	1	2	2	1	2	1	2	2	1	2	1
$\#C_1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\#C_2$			1	1	2	2	3	4	4	5	5	6	7	7	8	
$\#C_3$				1	2	3	5	9	10	14	15	20	27	28	35	
$\#C_4$					2	3	7	14	19	30	34	50	75	83	112	
$\#C_5$						3	7	19	28	53	65	103	170	200	292	
$\#C_6$							7	19	37	76	108	186	331	416	651	
$\#C_7$								19	37	99	151	299	577	768	1288	
$\#C_8$									37	99	194	412	908	1293	2302	
$\#C_9$										99	194	525	1239	1991	3792	
$\#C_{10}$											194	525	1570	2689	5758	
$\#C_{11}$												525	1570	3387	7724	
$\#C_{12}$													1570	3387	9690	
$\#C_{13}$														3387	9690	
$\#C_{14}$															9690	

The values of the  $\#C_i$  meet a specific algorithm that starts at  $v = 4$ . We first give some examples before moving on to the general algorithm (only the values in red font are detailed again):

$v = 4, \Delta w = 2, \Delta w_{\text{prec1}} = 1, \#(v)=3$

1	1	1
1	1	1

$v = 5, \Delta w = 1, \Delta w_{\text{prec1}} = 2, \#(v)=7$

1	1	1	1
0	1	1	1
1	2	2	2

$v = 6, \Delta w = 2, \Delta w_{\text{prec1}} = 1, \#(v) = 12$

1	1	1	1	1
0	1	2	2	2
1	2	3	3	3

$v = 7, \Delta w = 2, \Delta w_{\text{prec1}} = 2, \#(v) = 30$

1	1	1	1	1	1
0	1	2	3	3	3
0	1	2	3	3	3
1	3	5	7	7	7

$v = 8, \Delta w = 1, \Delta w_{\text{prec1}} = 2, \#(v) = 85$

1	1	1	1	1	1	1
0	1	2	3	4	4	4
0	1	3	5	7	7	7
0	1	3	5	7	7	7
1	4	9	14	19	19	19

Let us have  $\#(v,i,j)$  an element of these tables (excluding last lines in red font). The number  $i$  is the index of line starting at 1 (downwards),  $j$  is the index column starting at 1 (rightwards),  $v$  is the number of the current iteration (and the number of odd-steps).

#### Proposition 4

The Pascal trihedron meets to the following rules :

$j_{\max} = v-1$
$\#(v,1,j) = 1, j = 1 \text{ to } j_{\max}-1$
$\#(v,i,1) = 0, i > 1$
$\#(v,i,j) = \#(v,i-1,j) + \#(v-1,i,j-1), i > 1, j > 1$
The last line is doubled if $\Delta w_{\text{prec1}}(v) = 2$ , thus incrementing the number of rows
Sum up for $\#(v) = \sum \#(v,i,j)$ all columns and rows of the table (or sum up the line in red)

#### Relationship with Pascal triangle.

This formula has a kinship with Pascal's triangle. However, the latter increases by sections of lines, when instead here the enlargement is made by sections of planes. The routine gives not a triangle but a Pascal trihedron. The edges of this trihedron are less smooth than those of Pascal's triangle, but they get this way with some hindsight.

The increase in the number of columns is 1 when the number of steps odd  $v$  is incremented by 1. Looking at lines, their number increases of 1 or 0 depending on the value of  $\Delta w_{\text{prec1}}(v)$ , thus the non-perfectly smooth edges of the Pascal trihedron.

#### Fibonacci series is never far away.

The evolution towards constant ratios for  $\#(v)/\#(v-1)$ , which we give an illustration below, has a relationship with Fibonacci series and Pascal's triangle.

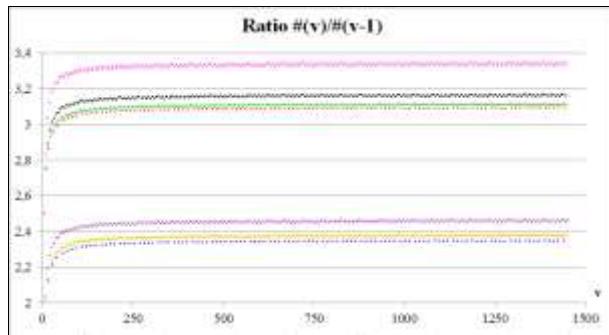
For the first item, we have the evolution :

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
#vfib(n)	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610
#vfib(n) / #vfib(n-1)		1	2	1,5	1,6667	1,6	1,625	1,6154	1,619	1,6176	1,6182	1,618	1,6181	1,618	1,618

It is well established matter that successive numbers of Fibonacci series ratio tends towards the golden ratio  $(1+\sqrt{5})/2$  and is therefore bounded. This is the result of a recursive application on two numbers.

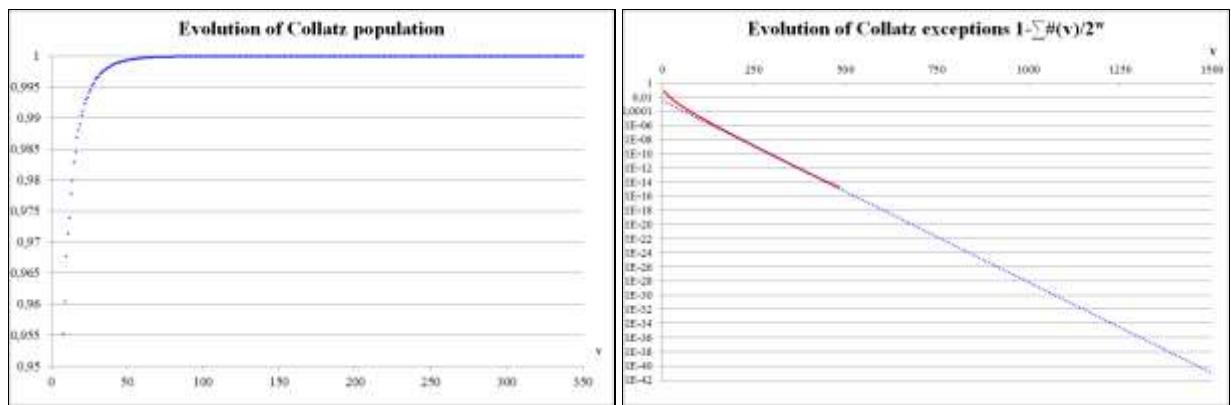
Turning to Pascal's triangle, with a routine where  $n$  numbers occur, where  $n$  increases by 1 at each step, the ratio  $\#vpas(n)/\#vpas(n-1)$ , where  $\#vpas(n)$  is the sum of the terms of a line of the said triangle, is equal to  $C(n,2)/C(n-1,2) = 2$  (where  $C(n,m)$  is the number of combinations of  $m$  elements among  $n$  elements).

Passing then to what we called a Pascal trihedron, the minimum ratio increases again and moreover splits up into several limit values.



#### Difference to the global population

Thanks to the values of  $\#(v)$  given in appendix 5, we can draw the two graphs below giving the relative proportions of the population of Collatz:



The first graph shows that between  $v = 0$  and  $v = 100$ , the bulk of the population of the natural numbers is reached.

The second graph reflects the evolution  $1 - \sum \#(v)/2^{w(v)}$  as new terms are added to the sum  $\sum \#(v)/2^{w(v)}$ . The proportion of the natural numbers  $N$  which does not meet the Collatz conjecture is therefore less than  $10^{-10}$  as soon as  $v = 300$ . The trend, with a linear abscissa  $v$  and a logarithmic ordinate axis, is a straight line (with enough hindsight).

As for reducing  $1 - \sum \#(v)/2^{w(v)}$ , it is now only matter of good hardware, appropriate software and computing time. Any takers?

#### References

- [1] <https://sites.google.com/site/schaetzelhubertdiophantien/> Article : Vol d'altitude avec les nombres de Collatz.  
La genèse d'un trièdre de Pascal
- [2] [http://fr.wikipedia.org/wiki/Conjecture\\_de\\_Syracuse](http://fr.wikipedia.org/wiki/Conjecture_de_Syracuse)
- [3] Riho Terras. A stopping time problem on the positive integers. *Acta Arithmetica* 30 (1976), 241–252

## APPENDIX 1

Sample of flight times and maximal flight altitudes.

N	Flight time	Flight time in altitude	Maximal flight attitude	Initial maximal flight attitude
2	1	1	2	2
3	7	6	16	16
4	2	1	4	4
5	5	3	16	16
6	8	1	16	6
7	16	11	52	52
8	3	1	8	8
9	19	3	52	28
10	6	1	16	10
11	14	8	52	52
12	9	1	16	12
13	9	3	40	40
14	17	1	52	14
15	17	11	160	160
16	4	1	16	16
17	12	3	52	52
18	20	1	52	18
19	20	6	88	88
20	7	1	20	20
21	7	3	64	64
22	15	1	52	22
23	15	8	160	160
24	10	1	24	24
25	23	3	88	76
26	10	1	40	26
27	111	96	9232	9232
28	18	1	52	28
29	18	3	88	88
30	18	1	160	30
31	106	91	9232	9232
32	5	1	32	32
33	26	3	100	100
34	13	1	52	34
35	13	6	160	160
36	21	1	52	36
37	21	3	112	112
38	21	1	88	38
39	34	13	304	304
40	8	1	40	40
41	109	3	9232	124
42	8	1	64	42
43	29	8	196	196
44	16	1	52	44
45	16	3	136	136
46	16	1	160	46
47	104	88	9232	9232
48	11	1	48	48
49	24	3	148	148
50	24	1	88	50
51	24	6	232	232
52	11	1	52	52
53	11	3	160	160
54	112	1	9232	54
55	112	8	9232	376
56	19	1	56	56
57	32	3	196	172
58	19	1	88	58
59	32	11	304	304
60	19	1	160	60

## APPENDIX 2

### Full list of associates of some modulo $2^w$ families

Family mod 2, v = 0  
0

Family mod 4, v = 1  
1

Family mod 16, v = 2  
3

Family mod 32, v = 3  
11, 23

Family mod 128, v = 4  
7, 15, 59

Family mod 256, v = 5  
39, 79, 95, 123, 175, 199, 219

Family mod 1024, v = 6  
287, 347, 367, 423, 507, 575, 583, 735, 815, 923, 975, 999

Family mod 4096, v = 7  
231, 383, 463, 615, 879, 935, 1019, 1087, 1231, 1435, 1647, 1703, 1787, 1823, 1855, 2031, 2203, 2239, 2351, 2587, 2591, 2907, 2975, 3119, 3143, 3295, 3559, 3675, 3911, 4063

Family mod 8192, v = 8  
191, 207, 255, 303, 539, 543, 623, 679, 719, 799, 1071, 1135, 1191, 1215, 1247, 1327, 1563, 1567, 1727, 1983, 2015, 2075, 2079, 2095, 2271, 2331, 2431, 2607, 2663, 3039, 3067, 3135, 3455, 3483, 3551, 3687, 3835, 3903, 3967, 4079, 4091, 4159, 4199, 4223, 4251, 4455, 4507, 4859, 4927, 4955, 5023, 5103, 5191, 5275, 5371, 5439, 5607, 5615, 5723, 5787, 5871, 5959, 5979, 6047, 6215, 6375, 6559, 6607, 6631, 6747, 6815, 6983, 7023, 7079, 7259, 7375, 7399, 7495, 7631, 7791, 7847, 7911, 7967, 8047, 8103

Family mod 32768, v = 9  
127, 411, 415, 831, 839, 1095, 1151, 1275, 1775, 1903, 2119, 2279, 2299, 2303, 2719, 2727, 2767, 2799, 2847, 2983, 3163, 3303, 3611, 3743, 4007, 4031, 4187, 4287, 4655, 5231, 5311, 5599, 5631, 6175, 6255, 6503, 6759, 6783, 6907, 7163, 7199, 7487, 7783, 8063, 8187, 8347, 8431, 8795, 9051, 9087, 9371, 9375, 9679, 9711, 9959, 10055, 10075, 10655, 10735, 10863, 11079, 11119, 11567, 11679, 11807, 11943, 11967, 12063, 12143, 12511, 12543, 12571, 12827, 12967, 13007, 13087, 13567, 13695, 13851, 14031, 14271, 14399, 14439, 14895, 15295, 15343, 15839, 15919, 16027, 16123, 16287, 16743, 16863, 16871, 17147, 17727, 17735, 17767, 18011, 18639, 18751, 18895, 19035, 19199, 19623, 19919, 20079, 20199, 20507, 20527, 20783, 20927, 21023, 21103, 21223, 21471, 21727, 21807, 22047, 22027, 22655, 22751, 22811, 22911, 22939, 23231, 23359, 23399, 23615, 23803, 23835, 23935, 24303, 24559, 24639, 24647, 24679, 25247, 25503, 25583, 25691, 25703, 25831, 26087, 26267, 26527, 26535, 27111, 27291, 27759, 27839, 27855, 27975, 28703, 28879, 28999, 29467, 29743, 29863, 30311, 30591, 30687, 30715, 30747, 30767, 30887, 31711, 31771, 31899, 32155, 32239, 32575, 32603

Family mod 65536, v = 10  
359, 479, 559, 603, 767, 859, 1179, 1183, 1351, 1519, 1535, 1627, 2367, 2407, 2495, 2671, 2687, 2791, 2887, 2927, 3103, 3239, 3487, 3535, 3695, 3815, 4319, 4335, 4379, 4635, 4775, 4799, 4815, 4895, 4991, 5087, 5343, 5375, 5423, 5583, 5663, 5823, 5863, 6207, 6247, 6555, 6639, 6703, 6975, 7015, 7103, 7231, 7451, 7471, 7551, 7711, 7835, 7871, 7931, 8095, 8263, 8551, 8671, 8863, 9119, 9199, 9319, 9543, 9599, 9819, 9935, 10151, 10559, 10727, 10907, 11035, 11247, 11431, 11727, 11823, 11887, 12007, 12319, 12495, 12615, 12775, 12799, 13279, 13339, 13535, 13615, 13671, 13855, 13927, 13951, 14015, 14207, 14303, 14363, 14383, 14503, 14543, 14747, 15103, 15167, 15207, 15423, 15487, 15515, 15599, 15643, 15743, 15771, 15855, 16191, 16411, 16431, 16455, 16511, 16635, 16831, 17055, 17127, 17135, 17223, 17311, 17391, 17479, 17511, 17659, 18159, 18343, 18523, 18559, 18919, 19099, 19111, 19135, 19151, 19231, 19367, 19547, 19687, 19707, 20127, 20207, 20511, 20591, 20687, 20807, 21039, 21595, 21615, 21695, 21735, 22015, 22119, 22399, 22495, 22555, 22575, 22695, 22887, 23143, 23167, 23583, 23663, 23707, 23711, 23743, 23963, 24047, 24383, 24571, 24703, 24731, 24815, 25371, 25415, 25471, 25599, 25671, 25851, 26015, 26063, 26343, 26351, 26367, 26439, 26459, 26619, 27039, 27119, 27303, 27343, 27423, 27559, 27675, 27739, 27879, 27903, 27951, 28095, 28191, 28319, 28327, 28351, 28447, 28507, 28527, 28927, 29087, 29231, 29631, 29807, 29823, 29887, 30079, 30207, 30235, 30415, 30575, 30655, 30971, 30975, 31079, 31199, 31335, 31359, 31471, 31727, 31775, 32223, 32283, 32303, 32703, 32763, 32859, 32923, 33007, 33087, 33255, 33531, 33663, 34111, 34151, 34255, 34271, 34535, 34631, 34651, 34927, 35023, 35231, 35279, 35311, 35419, 35579, 35583, 36143, 36159, 36383, 36519, 36543, 36635, 36639, 36719, 36891, 36911, 37119, 37167, 37311, 37407, 37467, 37487, 37607, 37735, 38047, 38171, 38271, 38427, 38607, 38847, 39039, 39135, 39195, 39295, 39535, 39615, 39919, 40039, 40187, 40351, 40415, 40495, 40687, 40943, 41023, 41063, 41183, 41243, 41447, 41627, 41723, 42075, 42215, 42239, 42303, 42343, 42471, 42651, 42911, 43071, 43111, 43215, 43335, 43471, 43611, 43775, 43967, 44143, 44223, 44239, 44359, 44699, 44959, 45083, 45103, 45223, 45359, 45503, 45535, 45599, 45679, 45799, 45851, 46127, 46247, 46407, 47099, 47231, 47327, 47387, 47423, 47487, 47807, 48095, 48155, 48295, 48379, 48879, 48987, 49135, 49215, 49255, 49311, 49563, 49567, 49983, 50143, 50267, 50303, 50407, 50663, 50843, 50847, 51055, 51103, 51271, 51431, 51451, 51455, 51611, 51871, 51951, 52031, 52071, 52335, 52415, 52431, 52507, 52551, 52735, 52763, 53159, 53183, 53319, 53339, 53439, 53887, 53919, 54043, 54303, 54319, 54375, 54439, 54751, 55207, 55291, 55327, 55407, 55535, 55963, 56059, 56191, 56287, 56315, 56347, 56639, 56935, 57179, 57215, 57375, 57671, 57755, 57759, 57839, 57947, 58175, 58203, 58495, 58523, 58527, 58863, 58983, 59247, 59263, 59463, 59559, 59623, 59643, 59647, 60015, 60063, 60143, 60231, 60271, 60571, 60831, 60911, 60955, 61135, 61351, 61375, 61531, 61631, 61663, 61723, 61979, 62119, 62159, 62239, 62279, 62719, 62943, 63023, 63335, 63519, 63551, 63591, 64047, 64167, 64207, 64251, 64287, 64447, 64507, 64831, 64871, 65127, 65179, 65183, 65275, 65407, 65439

**APPENDIX 3**  
List of smallest N and altitude flight times.

Smallest N	TFA	Even steps w	Odd steps v	Smallest N	TFA	Even steps w	Odd steps v	Smallest N	TFA	Even steps w	Odd steps v
2	1	1	0	10 087	171	105	66	29 256 191	342	210	132
5	3	2	1	256 511	174	107	67	8 837 211	344	211	133
3	6	4	2	67 583	176	108	68	2 091 647	347	213	134
11	8	5	3	90 111	179	110	69	1 394 431	349	214	135
7	11	7	4	45 055	181	111	70	17 392 879	352	216	136
39	13	8	5	126 575	184	113	71	13 002 751	355	218	137
287	16	10	6	299 259	187	115	72	7 460 635	357	219	138
231	19	12	7	96 383	189	116	73	2 533 535	360	221	139
191	21	13	8	336 199	192	118	74	1 689 023	362	222	140
127	24	15	9	64 255	194	119	75	1 126 015	365	224	141
359	26	16	10	84 383	197	121	76	64 993 051	368	226	142
511	29	18	11	57 115	200	123	77	19 925 503	370	227	143
239	32	20	12	56 255	202	124	78	13 774 695	373	229	144
159	34	21	13	37 503	205	126	79	9 280 639	375	230	145
639	37	23	14	60 975	207	127	80	46 043 247	378	232	146
283	39	24	15	45 127	210	129	81	28 290 175	380	233	147
991	42	26	16	393 967	212	130	82	57 330 463	383	235	148
251	44	27	17	423 679	215	132	83	54 870 655	386	237	149
167	47	29	18	1 759 951	218	134	84	46 355 695	388	238	150
111	50	31	19	35 655	220	135	85	48 773 915	391	240	151
1 695	52	32	20	434 223	223	137	86	32 515 943	393	241	152
1 307	55	34	21	495 687	225	138	87	41 946 879	396	243	153
871	57	35	22	665 215	228	140	88	12 132 095	399	245	154
927	60	37	23	1 643 759	231	142	89	8 088 063	401	246	155
671	63	39	24	528 895	233	143	90	21 677 295	404	248	156
155	65	40	25	730 559	236	145	91	14 378 779	406	249	157
103	68	42	26	437 247	238	146	92	41 942 559	409	251	158
1 639	70	43	27	2 162 111	241	148	93	241 682 847	412	253	159
91	73	45	28	432 923	243	149	94	40 814 363	414	254	160
3 431	75	46	29	565 247	246	151	95	187 375 615	417	256	161
3 399	78	48	30	288 615	249	153	96	131 801 135	419	257	162
2 287	81	50	31	376 831	251	154	97	44 186 399	422	259	163
71	83	51	32	2 548 479	254	156	98	29 457 599	424	260	164
6 395	86	53	33	611 455	256	157	99	39 276 799	427	262	165
47	88	54	34	608 111	259	159	100	19 638 399	430	264	166
31	91	56	35	1 585 403	262	161	101	53 271 551	432	265	167
2 047	94	58	36	405 407	264	162	102	71 028 735	435	267	168
27	96	59	37	270 271	267	164	103	27 209 575	437	268	169
1 819	99	61	38	362 343	269	165	104	35 514 367	440	270	170
17 691	101	62	39	401 151	272	167	105	60 112 511	443	272	171
6 887	104	64	40	1 563 647	275	169	106	40 075 007	445	273	172
4 591	106	65	41	1 042 431	277	170	107	53 433 343	448	275	173
13 439	109	67	42	6 721 703	280	172	108	143 061 311	450	276	174
6 383	112	69	43	381 727	282	173	109	500 752 231	453	278	175
4 255	114	70	44	667 375	285	175	110	162 612 223	455	279	176
7 963	117	72	45	626 331	287	176	111	107 295 983	458	281	177
7 527	119	73	46	1 691 807	290	178	112	22 649 071	461	283	178
12 399	122	75	47	1 564 063	293	180	113	71 530 655	463	284	179
7 279	125	77	48	1 541 147	295	181	114	20 132 507	466	286	180
1 583	127	78	49	1 027 431	298	183	115	13 421 671	468	287	181
1 055	130	80	50	1 127 871	300	184	116	401 306 907	471	289	182
703	132	81	51	1 991 615	303	186	117	279 200 511	474	291	183
15 039	135	83	52	1 327 743	306	188	118	20 638 335	476	292	184
111 259	138	85	53	7 303 711	308	189	119	272 473 947	479	294	185
41 407	140	86	54	6 255 855	311	191	120	757 916 519	481	295	186
62 079	143	88	55	6 492 187	313	192	121	836 710 559	484	297	187
77 031	145	89	56	7 849 755	316	194	122	26 716 671	486	298	188
94 959	148	91	57	3 137 471	318	195	123	144 091 295	489	300	189
34 239	150	92	58	9 294 427	321	197	124	192 121 727	492	302	190
138 751	153	94	59	8 484 287	324	199	125	96 060 863	494	303	191
99 007	156	96	60	2 788 863	326	200	126	64 040 575	497	305	192
106 239	158	97	61	7 499 935	329	202	127	340 208 287	499	306	193
187 327	161	99	62	6 079 559	331	203	128	56 924 955	502	308	194
69 375	163	100	63	6 204 543	334	205	129	525 068 415	505	310	195
226 767	166	102	64	20 808 639	337	207	130	431 557 735	507	311	196
104 303	169	104	65	9 941 863	339	208	131	...	...	...	...

## APPENDIX 4

$v = 4, \Delta w_{\text{prec1}} = 1, \#(v)=3$

1	1	1
1	1	1

$v = 5, \Delta w_{\text{prec1}} = 2, \#(v)=7$

1	1	1	1
0	1	1	1
1	2	2	2

$v = 6, \Delta w_{\text{prec1}} = 1, \#(v)=12$

1	1	1	1	1
0	1	2	2	2
1	2	3	3	3

$v = 7, \Delta w_{\text{prec1}} = 2, \#(v)=30$

1	1	1	1	1	1
0	1	2	3	3	3
0	1	2	3	3	3
1	3	5	7	7	7

$v = 8, \Delta w_{\text{prec1}} = 2, \#(v)=85$

1	1	1	1	1	1	1
0	1	2	3	4	4	4
0	1	3	5	7	7	7
0	1	3	5	7	7	7
1	4	9	14	19	19	19

$v = 9, \Delta w_{\text{prec1}} = 1, \#(v)=173$

1	1	1	1	1	1	1	1
0	1	2	3	4	5	5	5
0	1	3	6	9	12	12	12
0	1	4	9	14	19	19	19
1	4	10	19	28	37	37	37

$v = 10, \Delta w_{\text{prec1}} = 2, \#(v)=476$

1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	6	6
0	1	3	6	10	14	18	18	18
0	1	4	10	19	28	37	37	37
0	1	4	10	19	28	37	37	37
1	5	14	30	53	76	99	99	99

$v = 11, \Delta w_{\text{prec1}} = 1, \#(v)=961$

1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	7
0	1	3	6	10	15	20	25	25
0	1	4	10	20	34	48	62	62
0	1	5	14	30	53	76	99	99
1	5	15	34	65	108	151	194	194

$v = 12, \Delta w_{\text{prec1}} = 2, \#(v) = 2652$

1	1	1	1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	8	8	8	8
0	1	3	6	10	15	21	27	33	33	33	33
0	1	4	10	20	35	55	75	95	95	95	95
0	1	5	15	34	65	108	151	194	194	194	194
0	1	5	15	34	65	108	151	194	194	194	194
1	6	20	50	103	186	299	412	525	525	525	525

$v = 13, \Delta w_{\text{prec1}} = 2, \#(v) = 8045$

1	1	1	1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	8	9	9	9
0	1	3	6	10	15	21	28	35	42	42	42
0	1	4	10	20	35	56	83	110	137	137	137
0	1	5	15	35	69	121	191	261	331	331	331
0	1	6	20	50	103	186	299	412	525	525	525
0	1	6	20	50	103	186	299	412	525	525	525
1	7	27	75	170	331	577	908	1239	1570	1570	1570

$v = 14, \Delta w_{\text{prec1}} = 1, \#(v) = 17637$

1	1	1	1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	8	9	10	10
0	1	3	6	10	15	21	28	36	44	52	52
0	1	4	10	20	35	56	84	119	154	189	189
0	1	5	15	35	70	125	205	310	415	520	520
0	1	6	21	55	120	228	391	609	827	1045	1045
0	1	7	27	75	170	331	577	908	1239	1570	1570
1	7	28	83	200	416	768	1293	1991	2689	3387	3387

$v = 15, \Delta w_{\text{prec1}} = 2, \#(v) = 51033$

1	1	1	1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	8	9	10	11
0	1	3	6	10	15	21	28	36	45	54	63
0	1	4	10	20	35	56	84	120	164	208	252
0	1	5	15	35	70	126	209	325	474	623	772
0	1	6	21	56	125	246	437	716	1083	1450	1817
0	1	7	28	83	200	416	768	1293	1991	2689	3387
0	1	7	28	83	200	416	768	1293	1991	2689	3387
1	8	35	112	292	651	1288	2302	3792	5758	7724	9690

$v = 16, \Delta w_{\text{prec1}} = 1, \#(v) = 108950$

1	1	1	1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	8	9	10	11
0	1	3	6	10	15	21	28	36	45	55	65
0	1	4	10	20	35	56	84	120	165	219	273
0	1	5	15	35	70	126	210	329	490	693	896
0	1	6	21	56	126	251	456	766	1206	1776	2346
0	1	7	28	84	209	451	872	1534	2499	3767	5035
0	1	8	35	112	292	651	1288	2302	3792	5758	7724
1	8	36	119	322	753	1563	2946	5096	8207	12279	16351

$v = 17, \Delta w_{\text{prec1}} = 2, \#(v) = 312455$

1	1	1	1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	8	9	10	11
0	1	3	6	10	15	21	28	36	45	55	66
0	1	4	10	20	35	56	84	120	165	220	285
0	1	5	15	35	70	126	210	330	494	710	978
0	1	6	21	56	126	252	461	786	1260	1916	2754

0	1	7	28	84	210	461	912	1658	2794	4415	6521	8627	10733	10733	10733
0	1	8	36	119	322	753	1563	2946	5096	8207	12279	16351	20423	20423	20423
0	1	8	36	119	322	753	1563	2946	5096	8207	12279	16351	20423	20423	20423
1	9	44	156	448	1106	2429	4829	8831	14960	23741	35174	46607	58040	58040	58040

v = 18,  $\Delta w_{prec1} = 1$ , #(v)=663535

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	14	14
0	1	3	6	10	15	21	28	36	45	55	66	78	90	102	102	102
0	1	4	10	20	35	56	84	120	165	220	286	363	440	517	517	517
0	1	5	15	35	70	126	210	330	495	714	996	1341	1686	2031	2031	2031
0	1	6	21	56	126	252	462	791	1281	1974	2912	4095	5278	6461	6461	6461
0	1	7	28	84	210	462	923	1703	2939	4768	7327	10616	13905	17194	17194	17194
0	1	8	36	120	329	784	1676	3266	5885	9864	15534	22895	30256	37617	37617	37617
0	1	9	44	156	448	1106	2429	4829	8831	14960	23741	35174	46607	58040	58040	58040
1	9	45	164	486	1239	2814	5820	11084	19651	32566	50874	74575	98276	121977	121977	121977

v = 19,  $\Delta w_{prec1} = 2$ , #(v)=1900470

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	15
0	1	3	6	10	15	21	28	36	45	55	66	78	91	104	117	117
0	1	4	10	20	35	56	84	120	165	220	286	364	454	544	634	634
0	1	5	15	35	70	126	210	330	495	715	1000	1360	1795	2230	2665	2665
0	1	6	21	56	126	252	462	792	1286	1996	2974	4272	5890	7508	9126	9126
0	1	7	28	84	210	462	924	1715	2989	4935	7742	11599	16506	21413	26320	26320
0	1	8	36	120	330	791	1708	3391	6255	10820	17606	27133	39401	51669	63937	63937
0	1	9	45	164	486	1239	2814	5820	11084	19651	32566	50874	74575	98276	121977	121977
0	1	10	54	164	486	1239	2814	5820	11084	19651	32566	50874	74575	98276	121977	121977
1	10	54	210	658	1764	4193	9052	18033	33413	58054	94818	146567	213301	280035	346769	346769

v = 20,  $\Delta w_{prec1} = 2$ , #(v)=5936673

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	1	3	6	10	15	21	28	36	45	55	66	78	91	105	119	133
0	1	4	10	20	35	56	84	120	165	220	286	364	455	559	663	767
0	1	5	15	35	70	126	210	330	495	715	1001	1364	1815	2354	2893	3432
0	1	6	21	56	126	252	462	792	1287	2001	2997	4338	6087	8244	10401	12558
0	1	7	28	84	210	462	924	1716	3002	4990	7932	12080	17686	24750	31814	38878
0	1	8	36	120	330	792	1715	3424	6393	11245	18752	29686	44819	64151	83483	102815
0	1	9	45	165	494	1278	2954	6238	12213	22329	38403	62252	95693	138726	181759	224792
0	1	10	54	210	658	1764	4193	9052	18033	33413	58054	94818	146567	213301	280035	346769
0	1	11	65	273	915	2602	6522	14771	30769	59676	108392	185557	299811	459794	665506	871218
1	11	66	285	987	2907	7548	17706	38168	76583	144333	256533	432031	689675	1048313	1507945	1967577

v = 21,  $\Delta w_{prec1} = 1$ , #(v)=13472296

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	1	3	6	10	15	21	28	36	45	55	66	78	91	105	120	135
0	1	4	10	20	35	56	84	120	165	220	286	364	455	560	679	798
0	1	5	15	35	70	126	210	330	495	715	1001	1365	1819	2375	3033	3691
0	1	6	21	56	126	252	462	792	1287	2002	3002	4362	6157	8462	11277	14092
0	1	7	28	84	210	462	924	1716	3003	5004	7992	12294	18237	26148	36027	45906
0	1	8	36	120	330	792	1716	3431	6427	11397	19237	31046	47923	70967	100178	129389
0	1	9	45	165	495	1286	2994	6385	12665	23610	41566	69449	110175	166660	238904	311148
0	1	10	55	219	705	1944	4758	10578	21717	41643	74979	127503	204993	313227	452205	591183
0	1	11	65	273	915	2602	6522	14771	30769	59676	108392	185557	299811	459794	665506	871218
1	11	66	285	987	2907	7548	17706	38168	76583	144333	256533	432031	689675	1048313	1507945	1967577

v = 22,  $\Delta w_{\text{prec1}} = 2$ , #(v)=39993895

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	1	3	6	10	15	21	28	36	45	55	66	78	91	105	120	136	152	168
0	1	4	10	20	35	56	84	120	165	220	286	364	455	560	680	815	950	1085
0	1	5	15	35	70	126	210	330	495	715	1001	1365	1820	2379	3055	3848	4641	5434
0	1	6	21	56	126	252	462	792	1287	2002	3003	4367	6182	8536	11517	15125	18733	22341
0	1	7	28	84	210	462	924	1716	3003	5005	8007	12359	18476	26773	37665	51152	64639	78126
0	1	8	36	120	330	792	1716	3432	6434	11432	19404	31596	49522	74696	108632	151330	194028	236726
0	1	9	45	165	495	1287	3002	6426	12819	24097	43014	73162	118971	184871	275292	390234	505176	620118
0	1	10	55	220	714	1992	4946	11184	23397	45814	84657	148141	246474	389864	588519	842439	1096359	1350279
0	1	11	66	285	987	2907	7548	17706	38168	76583	144333	256533	432031	689675	1048313	1507945	1967577	2427209
0	1	11	66	285	987	2907	7548	17706	38168	76583	144333	256533	432031	689675	1048313	1507945	1967577	2427209
1	12	77	352	1285	3975	10809	26476	59457	123991	242517	448116	784511	1306067	2067149	3122122	4470986	5819850	7168714

v = 23,  $\Delta w_{\text{prec1}} = 1$ , #(v)=87986917

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	1	3	6	10	15	21	28	36	45	55	66	78	91	105	120	136	153	170
0	1	4	10	20	35	56	84	120	165	220	286	364	455	560	680	816	968	1120
0	1	5	15	35	70	126	210	330	495	715	1001	1365	1820	2380	3059	3871	4816	5761
0	1	6	21	56	126	252	462	792	1287	2002	3003	4368	6187	8562	11595	15388	19941	24494
0	1	7	28	84	210	462	924	1716	3003	5005	8008	12375	18546	27038	38368	53053	71093	89133
0	1	8	36	120	330	792	1716	3432	6435	11439	19440	31779	50142	76560	113064	161685	222423	283161
0	1	9	45	165	495	1287	3003	6434	12861	24258	43537	74793	123304	195531	297935	436977	612657	788337
0	1	10	55	220	715	2001	4995	11380	24045	47655	89351	159450	271445	442005	687799	1025496	1455096	1884696
0	1	11	66	286	1000	2988	7902	18928	41751	85823	165934	303783	527978	874036	1377474	2073809	2963041	3852273
0	1	12	77	352	1285	3975	10809	26476	59457	123991	242517	448116	784511	1306067	2067149	3122122	4470986	5819850
1	12	78	363	1353	4287	11967	30141	69653	149554	301174	573155	1036484	1784493	2932859	4597259	6893370	9821192	12749014

## APPENDIX 5

### List of associates' staffs

v	#(v)	Nb digits
1	1	1
2	1	1
3	2	1
4	3	1
5	7	1
6	12	2
7	30	2
8	85	2
9	173	3
10	476	3
11	961	3
12	2652	4
13	8045	4
14	17637	5
15	51033	5
16	108950	6
17	312455	6
18	663535	6
19	1900470	7
20	5936673	7
21	13472296	8
22	39993895	8
23	87986917	8
24	257978502	9
25	820236724	9
26	1899474678	10
27	5723030586	10
28	12809477536	11
29	38036848410	11
30	84141805077	11
31	248369601964	12
32	794919136728	12
33	1857112329035	13
34	5636545892795	13
35	12732900345928	14
36	38088111350198	14
37	123110229387834	15
38	290838337577435	15
39	889949312454085	15
40	2029460152095008	16
41	6113392816333320	16
42	13759389839553008	17
43	41156292958100112	17
44	133180667145777072	18
45	315356241137505268	18
46	967303800643232882	18
47	2213388970068123188	19
48	6687324379116300569	19
49	21797112395398269352	20
50	52028134169251235063	20
51	160509643506854706934	21
52	369707873749224505928	21
53	1122428422670255691408	22
54	3672921591387837707209	22
55	8808298119720364971552	22
56	27272844922266198818078	23
57	63092460692093312467525	23
58	192189781828748623023765	24
59	438474769118020519475109	24
60	1325438036712274130536314	25
61	4327322846731848749589802	25
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258 <sub>8</sub>	8848125912045198184895076967065446228950747455278649514203127572070731529879940818984145753121273130228403223	113
259 <sub>23</sub>	2087640542702786808907897622469817050189808071071005376654358295576485249832528150614429074058414484162444373558	114
260 <sub>48</sub>	6463711260755653871256917831187731350001985133156955061829000158561691555319203016558042849664997820397878751888	114
261 <sub>338</sub>	2151790901621292548629528209284721175032985306829096456295051571633745232596382125775498849826405260242311631487	115
262 <sub>683</sub>	5273966226389324047704833581985213154043308028561741293347574196684386717532723701888051052562165447839570458257	115
263 <sub>4210</sub>	1662413527334155601931767945873944845681164451194558027941820274329672749673440981332477837537419681001344488184	116
264 <sub>2220</sub>	3933005867041439023561955309623285315324981312138149708250897143459350423279620364357414148583729264259336556161	116
265 <sub>97028</sub>	1219967385908006514627589587720891374925651228369886909993831266641300193250458029775699424381628030648060839564	117
266 <sub>06018</sub>	4066973072447478336438348591578181712088616035055546570330449764725959211444314393873872935321489597671781927171	117
267 <sub>80211</sub>	9983650826645826623628195196768769460061426414871688352875760026751475269888663510795621528550111592973479313041	117
268 <sub>899467</sub>	31505490010979521991926091572678167603910562966255219064226697862121722756582384761455368700426620148462511700	118
269 <sub>550863</sub>	74633620571825951883941392227369506641329786129713871169166674650000856317631199598367328224823348765310223321	118
270 <sub>5681892</sub>	231723085715688875845452764648425786601025027748134600503971973579269836211448440185033513210971518390417140474	119
271 <sub>8873668</sub>	5413546921999465663167992107185067654063127596273611422073215531931512971137295084556252041587639653939882126625	119
272 <sub>39527056</sub>	16677409941322400314430933039425198844970177717627038070614220381939382209078288093861426364688806422570571344	120
273 <sub>34012897</sub>	5534813680395062042016544804970778430537296727387071590227668385413330375543295863450816190114839643291754890632	120
274 <sub>302721892</sub>	135256331559097679069744355728137832470318126339253705557883674059883946380375885781800083359980605011220731562	121
275 <sub>616843099</sub>	4255786667433295143843136829902330554189556630342516215181764696433596486005023564818980646447430388380540815552	121
276 <sub>1316168465</sub>	100507965133593724270691700117907364032493281348267090074424781907270062957411183724562097608250206520987430919	122
277 <sub>0041927655</sub>	311423171833494652425862247335775617887821974521065125582263351573892246045220227374830265501138121069290631022	122
278 <sub>5042261784</sub>	10373937643898329714776822926471557293048255043904846738227389868470143009416632816495865383735757873172618676	123
279 <sub>69237245649</sub>	254459867600531686464887590255712591470818216654726999565166664177623056319765023797555252360222411670452836388	123
280 <sub>10675673357</sub>	802586132112894336761481197589840739129936922268414411981338955272486127041444939730239339665170645319586027059	123
281 <sub>283201963409</sub>	1900243208391812283081015328835893904952708898132898203459261129301290408677559709991913103147375486387351078976	124
282 <sub>185759029517</sub>	58978793314513285618986444624726787331401922178218005965146559925933578132198696115708518626819127766245561363	124
283 <sub>6094433445282</sub>	1377395465910658131307504268490471399227514033683549190443794331376905518302236186059442337315781814516679466163	125
284 <sub>5690369697614</sub>	424240361976613221405473675426761556434541102046368322610686789482208656958958378306995109238209884240186810521	125
285 <sub>30921987756487</sub>	1407740465248804271465877069117170883544311220205046377938064124416661695984550040390640973039665509733512353483	126
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287 <sub>523908465240701</sub>	10821780532761342280467046647564695744820690100155817351429690443275051367067437662630119182265527128317719783	127
288 <sub>50299435049063</sub>	2555560318493992163598067550879844792205692903858456698221757157982092476661504489428412080264185453240099077437	127
289 <sub>505596877793260</sub>	7918100779422726827788217883056448669721263837107724933002236674191659636007263983860606735929814498735366203524	127
290 <sub>4567078773591813</sub>	2637593789899524184896618497170622729733426596145566181000137071454736075308619226887265252012590931986218981737	128
291 <sub>106959120480378</sub>	64696972356942418837721768975745378617674139266695541401657424560274977309029955129459198602866794904132629806	128
292 <sub>84097863661804954</sub>	2040626454115722817426154129561755313116607592323878722071114684214157861938863171992186057131131569232362495143	129
293 <sub>94069569749469003</sub>	4831666139051982892228195736600224690879874351544685155700242788691851945121698049274736781505173609359312655441	129
294 <sub>909259277306886700</sub>	149969274786289644530731682301906942245053952021601180539592276022845475479690499272393122427249075159462374	130
295 <sub>375120450251708522</sub>	350262624704380997428172470249301691459517369252892341215501358375698993552284091733297299818146764162662513637	130
296 <sub>4200286802586036812</sub>	107888708783593185848667004482030028089446633153882389645128306427241155055369675802045004742090093878141820935	131
297 <sub>1766021520331674689</sub>	35802623366612210259305098201763207892902980470613292271578424202848820410481673686038247400256808308261699474	131
298 <sub>8210165688393711707</sub>	874874235086905554352721214816484684314524229009474472165258087316612412839438067594717136074837369417127007160	131
299 <sub>30449933121898436671</sub>	2752744606402545366473274471332487946209773017851589872573320020259450701215631402507203459936089959921765703306	132
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