

## **Guide for the articles of this Web site**

(To contact the author : [hubert.schaetzel@wanadoo.fr](mailto:hubert.schaetzel@wanadoo.fr))

This site is the result of an accidentally reading around 1995 in a mathematics popularizing work. Formulas and geometrical figures chosen for their elegance or other more basic reasons were described. It turns out that one of them was the Goldbach conjecture. As many, I'd never realized that the field of mathematical research still was an open topic for apparently so obvious problems.

In one of his interviews, Alain Connes summarizes with great common sense and discernment what matters to the mathematical discovery, in substance, to "be stumped" long enough on a subject. This was my case having very little ideas for quite a long time. But afterwards, having written quite a lot, this site is here in the hope that the material produced will be resumed and exploited at least in part.

It is not a Math class, nor a synthesis of some existing mathematical work. It all started as a game and essentially lucky findings have "fallen from heaven". Everyone is delighted first by its own small successes.

This being said, why resorting to a personal Web site instead of publications via an "official" channel? Let us say that it is not for lack of trying. We remember here the caricature example of Alexandre Grothendieck in *Récoltes et Semailles* of "this of yesteryear's friend... who did accept to submit a note to the 'comptes rendus' only when the stated outcomes were a full surprise or that it happened that he couldn't prove them by himself..." The remainder of this extract is much more sharply-worded...

The purpose of mathematics is certainly to follow a pitch and produce a proof from A to Z. We are not less ambitious, but some problems are known to be hard-wearing. Thus, we had sometimes (and even often) to be satisfied with the middle of the alphabet and to choose instead of the word proof, the terms of "method" for some articles and "arithmetic contributions" as Web site's header. Let us note also that there is certainly the possibility of rewriting some articles, especially older ones, in a more condensed way.

Let us give now some axis for reading.

Even if the articles are sufficiently independent to jump from one subject to another, we suggest the following order,

- group 1, articles 1 to 6
- group 2, article 7 (in fact two articles)
- group 3, articles 8 to 11
- group 4, articles 12 to 13 (Millennium challenges)

The articles' contents are given underneath :

## 1. Principle of Hasse and obstructions

The goal is to give a different perspective to the usual notion of obstructions to the Hasse principle. The raw assessment of obstruction in an equation is replaced by the comparison with a family of similar equations showing that this notion has very varying degrees of manifestation up to showing just the opposite sometimes (i.e. affluence). We show that an "obstruction" corresponds to the possibility of rewriting the given equation. The fundamental concept of "asymptotic representative" or "local variable" introduced in this article remains the core of the following article.

## 2. Asymptotic diophantine enumerations. Condensed concepts.

In thirty pages, we try to summarize the essential on asymptotic (therefore with an infinite number of integer solutions) enumeration for diophantine equation, giving keys and clues. It is a condensed version of the underneath article with numerous points not disclosed again.

## 3. Asymptotic diophantine enumerations. Method of hypervolumes.

You want to enumerate prime numbers of the form  $x^3+x^2y+xy^2+y^3+5t^2+9u^4+4$ . Follow the method that will tell you how to do it. In particular, you will see how to compose the basic bricks of the problem : multiplication by a constant (here 5 and 9), treatment of monomials, treatment of  $x^3+x^2y+xy^2+y^3$  and then overall treatment. The final constant (here 4) will act in the form of conditions of divisibility and any other choice is treated identically. The choice of variables, either integers or prime numbers, is also free.

However, it will be necessary for the reader to get safely to the 'exercise' 9 of our article for that to become obvious. The astute reader will also see the wall of the problems raised by polynomials\*\*\* (compared with the monomials 'easy' case).

The method is based on two points :

- the substitution, to a given variable, by a mathematical equivalent object that we call its representative, the key point being the notion of equiprobability,
- the transcription of the calculations in a matrix form.

We show that the addition of a variable (or as the case of a grouping of imbricated variables independent of other variables such as  $x^3+x^2y+xy^2+y^3$ ) relates first to a multiplication by a matrix, then to a simple multiplication of eigenvalues, the base changing matrices being neutralized in intermediate steps. We believe that this is a new and innovative contribution in the field of the theory of numbers. Of course, the circulating matrices that are implemented show components like  $e^{i2\pi uv}$  recalling Gauss sums (so familiar in number theory, confirming somehow the consistency of the method...)

\*\*\* For those, featuring the finite sizes matrices addressed in our article in monomials' cases, we surmise the existence of square matrices of infinite size with eigenvalues sufficiently remarkable for their determination. Once discovered, the asymptotic enumeration method will extend de facto to combinations of polynomials, monomials, and symmetrical expressions using mere eigenvalues' products.

## 4. Arguments for a theorem of twin prime numbers.

To have a method for complicated equations (like above) does not provide effective proof for more trivial equations. Thus, we use a specific approach for the twin prime numbers case. Evidence of the infinity of the cardinal is flagrant. We use the Eratosthenes' sieve to prove it by deleted proportions considerations on the one hand and by assessment of maximum gaps on the other hand. We apply this sieve to prime numbers and to twin prime numbers. From one to the other cases, demonstration pillars are very similar, hence for proportions passing from  $(p-1)/p$  to  $(p-2)/p$  and for gaps changing  $2p$  to  $\sum 2p$ .

## 5. Goldbach conjecture : a typical statistical matter.

By selecting different sets of numbers, we show to some extent that the Goldbach conjecture is mainly a statistical matter.

## 6. Literal enumeration of the number of points on an elliptic curve in finite fields $\mathbb{Z}/p\mathbb{Z}$ .

Starting with another approach, we find on our way part of the above matrices (cf. method of hypervolumes), because, as we said earlier, everything is related in mathematics. We had also fun in realizing large numerical calculations (may be somewhat vain?)

## 7. Altitude flight with the Collatz numbers : the genesis of a Pascal trihedron + Collatz conjecture : Geography of the Pascal trihedron.

A three-dimensional sophisticated relative of the Pascal triangle governs the families (modulo  $2^w$ ) of numbers of equal stopping time subject to the Collatz algorithm. The first article (and its simplified version) provides main steps which lead to this discovery while limiting the subject to the enumeration of families' members.

The second article specifies the characteristics of the numbers of a given family and compares with others families, gives their geography in the Pascal trihedron and methods for anticipation.

## 8. Continued fractions of Dirichlet series. Recurrence polynomials study.

We give a Dirichlet series continued fraction expansion. That alone seemed us likely of interest but sadly refused for publication without explanation (no trace nevertheless of this expansion in mathematical tables available on the web). In addition, all zeros of the recurrence polynomials share the same constant real part ( $-1/2$ ), which definitely will remind us the Riemann conjecture. The continued fraction is another revealing object of that remarkable property of unicity.

## 9. Locus of the Riemann Zeta function zeroes.

Is this problem a problem of mathematics or of cosmology, a problem of black holes?

## 10. Imaginary values of Riemann Zeta function zeros

Some speculations are done on that subject.

## 11. The Siamese twins of the Zeta function zeros.

This is the Tom Thumb story within the Riemann hypothesis' framework. Indeed, by choosing to solve the Dirichlet series instead of the Riemann series (the Zeta-function), we get a supplementary set of zeros having proved systematic real value 1. Like the pebbles of the fable, they implicitly indicate the constant value (equal to  $1/2$ ) conjectured for the other zeros. In addition, for both zeros (of real values  $1/2$  and 1), we find more general common expressions of cancellation. So, these two sets adopt totally similar behavior, hence the title of the article.

## 12. Convexity in in lower half of the critical band and Riemann hypothesis

The evaluation of the lower boundary of a partial derivative extracted from Dirichlet's Eta function confirms Riemann's hypothesis.

### 13. Enquiry on the mainly stochastic nature of Birch and Swinnerton-Dyer's conjecture.

This is a most interesting problem combining Mertens' third theorem, the distribution of Satō-Tate and his complex multi-multiplication colleague, Kolyvagin-Gross-Zagier's calibration and other works. To see and see again.