

Are all integers Lychrel or Cheryl's ?

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Abstract When executing Wade VanLandingham's algorithm, palindromy, if it occurs, is not an invaluable achievement, but a mere youth event, a banal incident. The purpose of this article is to give a measure of the gap to palindromy and its evolution during the process in order to justify this cavalier assertion and at the same time show how similar all integers are when subjected to this algorithm. However, what initially seems to be an evidence in base 10 suddenly takes a different turn in base 2, risking to dispute the actual nature of integers in any base and leaving more questions than answers as summarized in the title of the article.

Les nombres entiers sont-ils tous de Lychrel ou de Cheryl ?

Résumé Lors de l'exécution de l'algorithme de Wade VanLandingham, la palindromie, si elle survient, n'est pas un aboutissement à valeur inestimable, mais un simple évènement de jeunesse, un incident banal. Le but de cet article est de donner une mesure de l'écart à la palindromie et son évolution au cours du processus afin de justifier cette affirmation cavalière et de montrer dans le même temps à quel point tous les nombres entiers se ressemblent lorsque soumis à cet algorithme. Cependant, ce qui semble d'abord être une évidence en base 10 prend soudain un tour différent en base 2 risquant de remettre en cause la nature effective des nombres dans toutes les bases et laissant plus de questions que de réponses comme le résume le titre de l'article.

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1.Introduction.

The title of the article is a provocative banter. Its meaning, quite elastic, expands or contracts, as this reading progresses.

Wade VanLandingham's algorithm consists of successive additions of a result with its symmetrical. If the operation does not result in a palindrome, the initial number is a Lychrel number. Few theoretical studies are available on this subject. Thus the main source and database used here are those of reference [1].

Years of continuous calculations have been carried out on certain numbers, including the presumed "first one" of them in order to confirm the absence of a palindrome. The research work thus seems to be mainly focused on the brute-force method of launching a program in the hope of setting a record, knowing that the only real record is infinity. Herein we propose to assess the gap with palindromy and gauge within a few tens of seconds the interest of conducting such an investigation. We will show that a massive and prolonged search is doomed to fail regardless of the initial number in the range of number sizes of current records (so for 196 also).

Much greater challenges however hide behind Lychrel which are exposed in paragraph 6.

2.Programming preliminaries.

The palindromy gap assessment is based on a VBA-based computer program that can be executed with a simple spreadsheet. The 0-gap does not stop the program, this event being considered a simple step of calculation of no real importance. The choices of initial number and number of steps of the process are set up at the beginning of the program provided in appendix 1.

The principle of calculation is explained below by an example.

So let us choose the integer 196. We get the succession of numbers of the second column by adding the symmetrical and examine its nature in relation to palindromy.

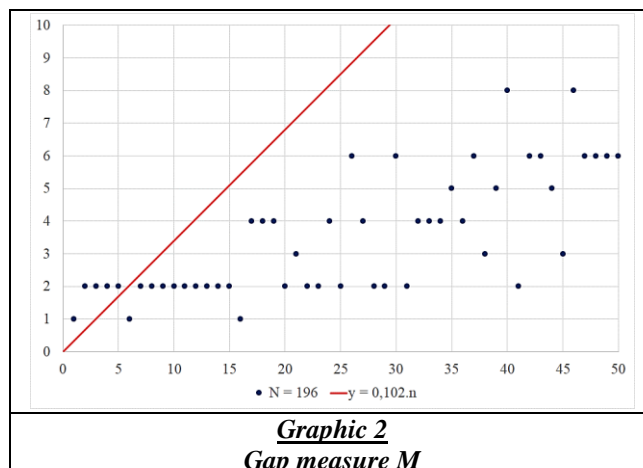
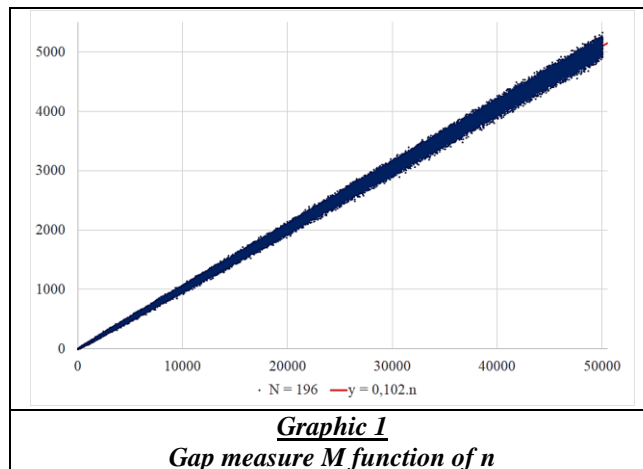
The term "thread" introduced by Jason Doucette corresponds to a sequence of numbers obtained by this process.

Steps n	Thread	Symmetric numbers	Associated palindromes	Gaps to palindrome	Measures M = Quantities of 1 within gap
0	196	691			
1	887	788	787	100	1
2	1675	5761	0575	1100	2
3	7436	6347	6336	1100	2
4	13783	38731	03773	10010	2
5	52514	41525	41514	11000	2
6	94039	93049	93039	01000	1
7	187088	880781	087078	100010	2
8	1067869	9687601	0967769	0100100	2
9	10755470	07455701	00745470	10010000	2
10	18211171	17111281	17111171	01100000	2
11	35322452	25422353	25322352	10000100	2
12	60744805	50844706	50744705	10000100	2

The palindrome associated with a given number is first determined by identifying the centre of symmetry of the examined number. Having done so, we then look for the smallest of the two numbers on either side of this centre and retain the smallest of these two which then serves as a substitute in the associated palindrome. A specific treatment of the numbers 1 and 9 is sometimes necessary and the reader will find it integrated into the algorithm attached in appendix 1.

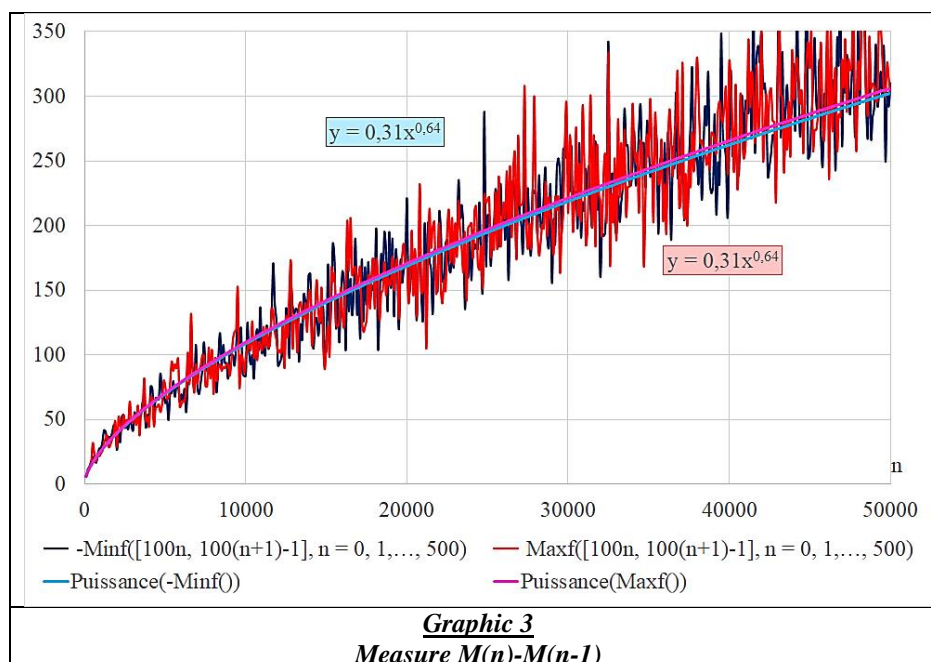
The reader will note that we are looking for a gap to the palindrome necessarily composed only of 0 or 1 and our algorithm is designed accordingly.

We then get the following two graphs, the second being a close-up view near the chart origin of the first one.



The number produced by the algorithm deviates from a palindrome by very few at steps 1, 6 and 16. It then departs following a cloud of points scattering around an average line $y = a.x$ where $a \approx 0.102$ ($x = n$, $y = M$). We call this middle line, the line of palindromic camber.

Let us have $M(n)$ the measure obtained at the iteration n . The graph below shows the evolution of positive or negative differences $M(n)-M(n-1)$. We take samples formed by a set of 100 iterations. For each set $\{100n, 100(n+1)-1\}$, we get a (negative) minimum and a (positive) maximum. The graph below gives the result of the 500 points provided in this way for the minimums (the sign - is added to the $\text{Min}()$ for comparison to the $\text{Max}()$) and 500 points also for the maximums.



We will return to the concern of $M(n)-M(n-1)$ gaps in paragraph 5 with more examples.

3.The line of palindromic camber.

Proposition

The line of palindromic camber is an invariant (in base 10). It is common to all integers.

Argument

The proposal is based on a very small number of trials, almost all of which are given in this article. Our conviction is strong even if the argument that follows remains rudimentary. However, it is based on the admission that there are no exceptional numbers in base 10 (see paragraph 6) which is by no means a trivial supposition. The gap to palindromy is weakly linked (hence the invariance) to the number initially chosen, but depends largely on the carries (of 1) of each 10-figure creating an ever-maintained chaos (hence the point cloud), the number of these carries being strongly linked to the length of the word (hence the average middle line) and the gap to palindromy is linked to the number of carries. Indeed, let us have L the length of the integer at step i . The algorithm acts on the digits 0 to 9 with have average of 4.5. For a large number of iterations, the addition would then increase the value of the number of a ratio $4.5/10 = 0.45$. Full-scale tests appear to indicate a slightly lower ratio of 0.41. The reader will find in appendix 2 a much more comprehensive study of the L/i ratio, as our argument is so far too simplistic.

Let us then have C the number of carries at step i . The probability of carries on the entire length of a large number with another whose digits are taken at random, not forgetting to add the carries each time, is 50%. Doing addition with mirror integers does not change this point if the distribution between digits 0 to 9 is homogenous for the current integer and the probability is the same for this particular algorithm, hence an asymptotic ratio for C/L equal to 0.5. This is true regardless of the base used.

Then let us look at the link between the number of carries and the number of deviations to the palindrome at step i . Let us take an example :

Initial integer	11692808688932019641372632460191398978072070
Mirror integer	07027087989319106423627314691023988680829611
Carries after addition	00100011111010011000000001100101111100110000
Addition	18719896678251126064999947151215387658901681
Palindrome	18619885678251115064999946051115287658891681
Gap to palindrome	00100011000000011000000001100100100000010000

A carry causes a deviation to the palindrome (highlight in red colour). This has a 1 to 1 effect in the absence of other effects. On the other hand, when carries are on either side of the number (highlight in blue colour), the two do not generate any effect. The relationship is 2 to 0. For the first type of event occurring twice as often as the second, the overall relationship is 2+2 versus 2+0, or 50%. There are of course other effects. For example, if all the digits are greater or equal to 5, the carries are systematic without destroying the palindromy. But this effect is transient and has no asymptotic consequences. Digital data show approximately a 50% equilibrium.

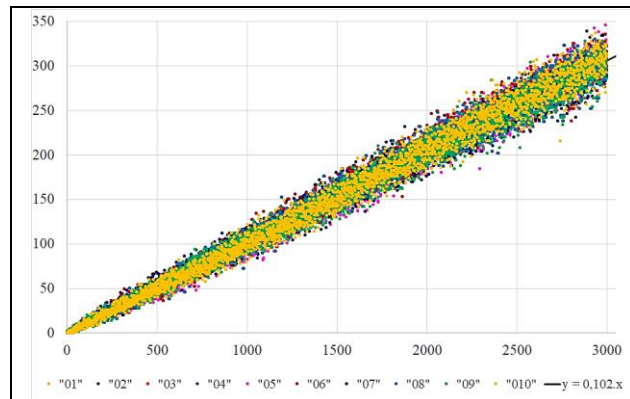
That said, the overall balance sheet then boils down to the proposal $M_{\text{average}} \approx (0.41).(0.5).(0.5).i \approx 0.102i$ which is effectively observed.

Note

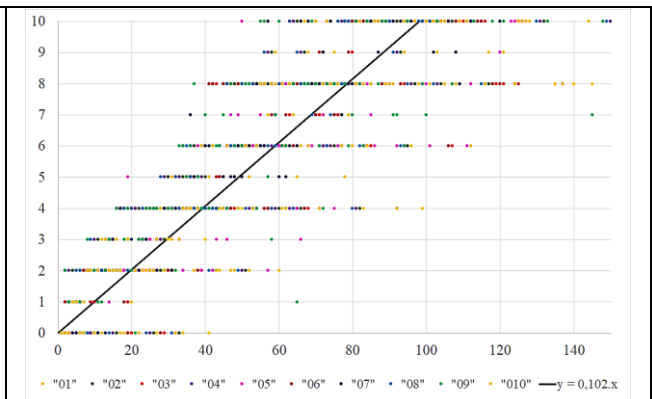
For very large numbers, the point cloud near the origin will obviously be shifted locally (with a positive ordinate at origin). But the asymptotic impact is totally negligible and the average middle line thus returns to its place when we take into account enough data.

Illustration

We chose the first ten integers : 01 to 010. The reader will distinguish those with the alternative choice of 1 to 10, very different, which would give several redundancies since 8, 4, 2 are twins to 1, as 6 to 3 and 10 to 5 in this specific algorithm framework.

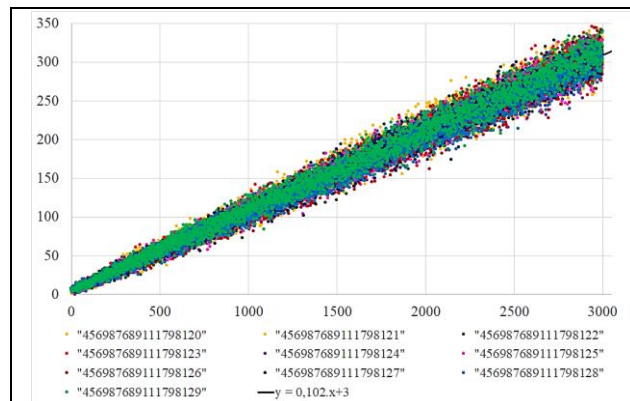


Graphic 4
Gap measure M

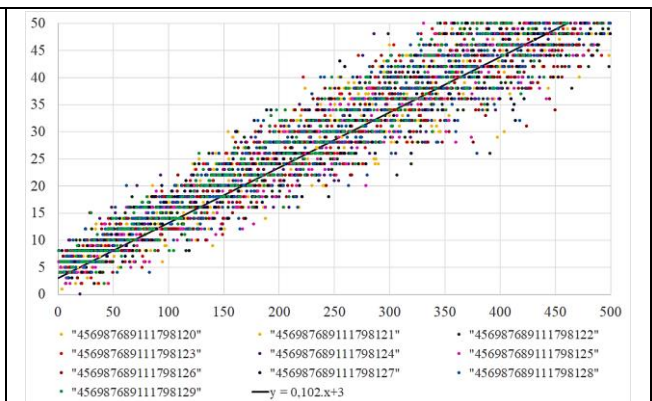


Graphic 5
Gap measure M

In addition (without any special precaution), we chose the ten numbers, with 18 digits, 456987689111798120 to 456987689111798129.



Graphic 6
Gap measure M



Graphic 7
Gap measure M

All these numbers have in common the same line of palindromic camber, the data being shifted at the graph origin in the second series of tests as shown in chart 7.

For 18 digits, there are statistically, according to Jason Doucette's website, 90.55% of Lychrel numbers. Here, the exception corresponding to this 10% is 45698768911798124 which has a 0-camber at step 21 (which here is only a happy coincidence).

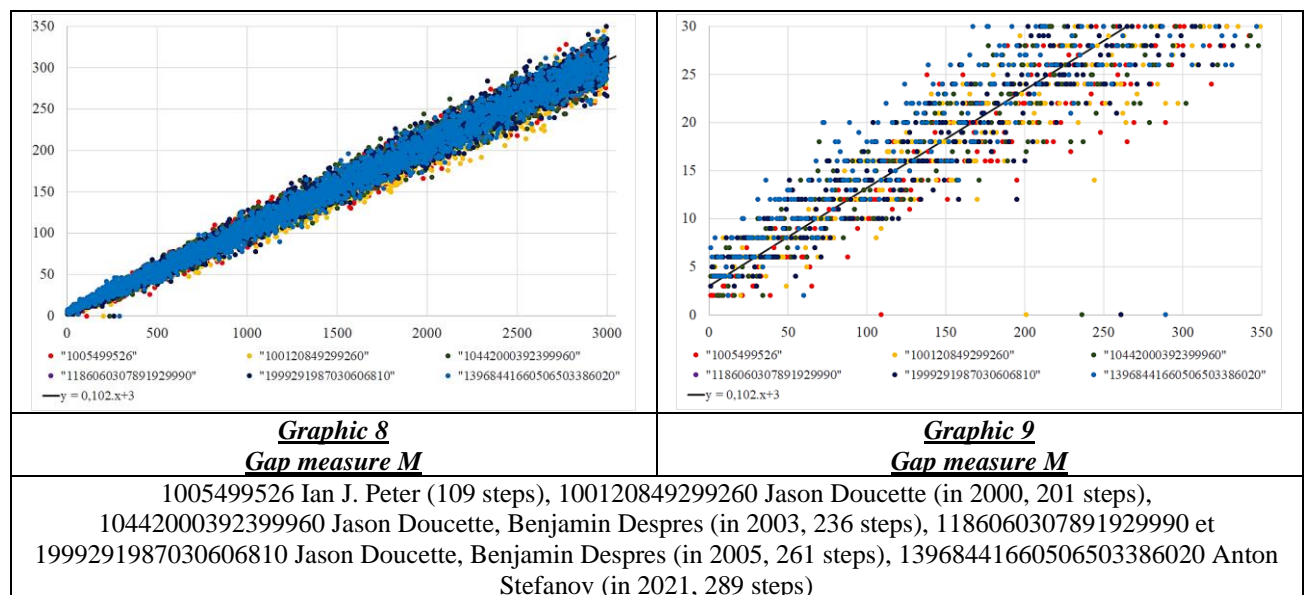
4.The smallest Lychrel number.

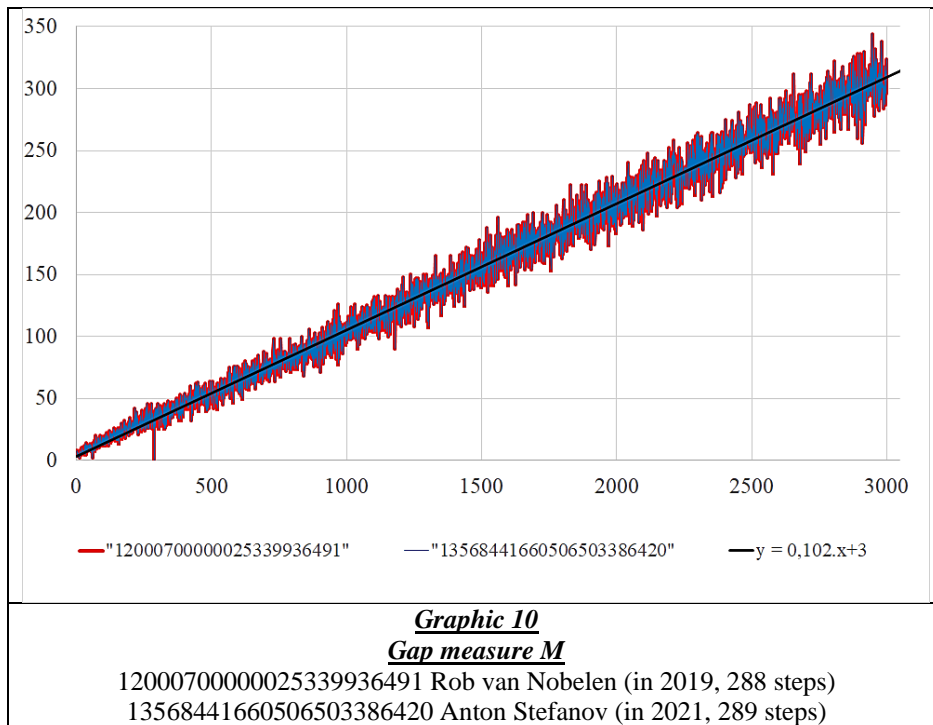
Usually, the smallest presumed Lychrel's number is cited as equal to 196. The reader is invited to open the territory to "097" which gives the same succession of numbers as "196", and is therefore his perfect smallest "twin":

Steps N	Thread	Symmetric numbers	Associated palindromes	Gaps to palindrome	Measures M Quantities of 1
0	097	790			
1	887	788	787	100	1
2	1675	5761	0575	1100	2
...

5.The atypical numbers.

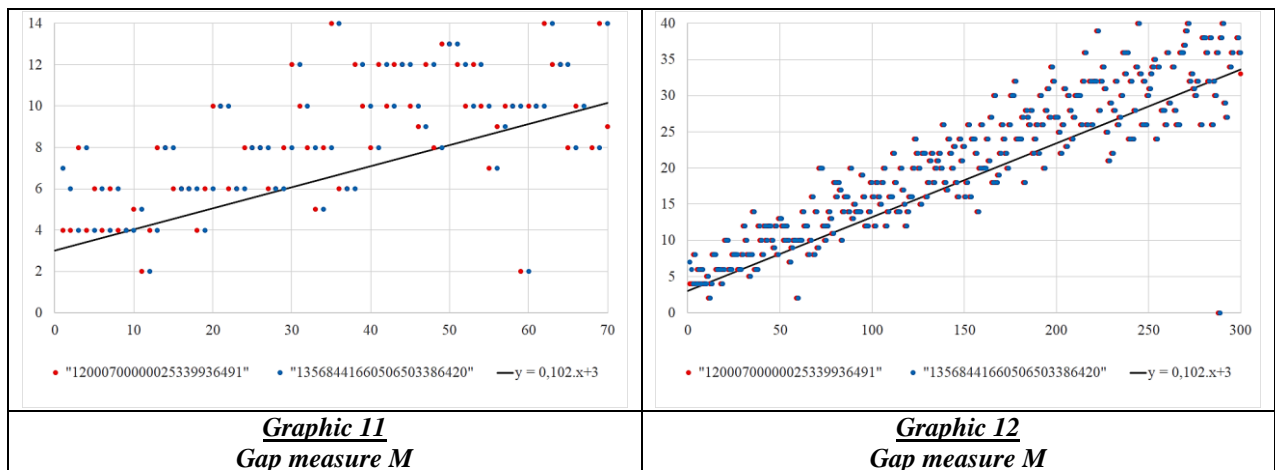
We name atypical those numbers whose particularity is to present a palindrome a little later than the others integers. The reader will be able to judge underneath by himself the extent of this deviation from the general pattern visible in the evolution of the point clouds subsequent to a 0-gap event.





For these records, the evolution of M at respective steps 288 and 289 is on either side $36 \searrow 0 \nearrow 38$. Although remarkable near origin, these changes are minor for more advanced iterations and become commonplace as suggested by another example (that of the initial number "196") given by graph 3. For this case quite standard, the first occurrence of more than -36 is at iteration 1132 ($132 \searrow 90 : -42$). The occurrence greater than or equal to +38 occurs at iteration 1390 ($128 \nearrow 166 : +38$). Over 3000 iterations, positive or negative differences of more than 40 are common. For Anton Stefanov's example, the first negative overtaking is at iteration 970 ($126 \searrow 78 : -48$) and the first positive catching-up at step 971 ($78 \nearrow 120 : +42$).

The graph below, a close-up view of the previous one, shows the subtle kinship of Rob van Nobelen and Anton Stefanov's records.



6.The exceptional numbers.

So far we looked at integers for which the linear evolution in size, number of carries and gap to palindromy was subject to a clear additional random part. For these, it seems that they all share the same growth speeds of the so-called size, number of carries and gap to palindromy around expected averages.

It turns out, however, that some numbers have totally predictable behaviours in relation to these three criteria (no random part). Panurge sheeps, they are totally different from their congeners, partially left to their misdirections. We do not know if they exist in base 10, but their presence is detected here in base 4 and 2. We call them Cheryl numbers or exceptional numbers.

Case 1 :

Base 4. Initial number : 2211101

Steps	Thread	Palindromes	Gap to palindrome	Sizes L	# Carries C	Measures M
0	221112	211112	10000	6		1
1	1032300	0032300	1000000	7	3	1
2	1131201	1021201	110000	7	3	2
3	2213112	2113112	100000	7	2	1
4	10332300	00322300	10010000	8	4	2
5	11322201	10222201	1100000	8	4	2
6	22211112	21111112	1100000	8	4	2
7	103323000	003222300	100100100	9	4	3
8	110312301	103212301	1100000	9	4	2
9	220131312	213131312	1000000	9	3	1
10	1033323000	0032222300	1001100100	10	5	4
11	1103222301	1032222301	11000000	10	5	2
12	2202111312	2131111312	11000000	10	5	2
13	10333230000	00322222300	10011001100	11	5	5
14	11003123301	10332123301	11000000	11	5	2
15	22001313312	21331313312	10000000	11	4	1
16	103333230000	003222222300	100111001100	12	6	6
17	110032223301	103322223301	110000000	12	6	2
18	220021113312	213311113312	110000000	12	6	2
19	1033332300000	0032222222300	1001110011100	13	6	7
20	1100031233301	1033321233301	110000000	13	6	2

Here the evolution of the initial integer follows a modulo 6 pattern. The values in brackets underneath correspond for the first bracket to the digit present in the studied number and for the latter bracket to the quantity of these identical in-a-row digits. For example, for $i = 18$, we have (2,2), (0,i/6-1), (2,1), (1,3), (3,i/6-1), (1,1), (2,1) which gives (2,2), (0,2), (2,1), (1,3), (3,2), (1,1), (2,1) and then (22), (00), (2), (111), (33), (1), (2) , or by putting things together 220021113312.

Steps i	Integers
6	22211112
12	2202111312
18	220021113312
$i = 0 \bmod 6$	(2,2),(0,i/6-1),(2,1),(1,3),(3,i/6-1),(1,1),(2,1)
1	1032300
7	103323000
13	10333230000
$i = 1 \bmod 6$	(1,1),(0,1),(3,(i-1)/6+1),(2,1),(3,1),(0,(i-1)/6+2)
2	1131201
8	110312301
14	11003123301
$i = 2 \bmod 6$	(1,2),(0,(i-2)/6),(3,1),(1,1),(2,1),(3,(i-2)/6),(0,1),(1,1)
3	2213112
9	220131312
15	22001313312
$i = 3 \bmod 6$	(2,2),(0,(i-3)/6),(1,1),(3,1),(1,1),(3,(i-3)/6),(1,1),(2,1)
4	10332300
10	1033323000
16	103333230000
$i = 4 \bmod 6$	(1,1),(0,1),(3,(i-4)/6+2),(2,1),(3,1),(0,(i-4)/6+2)
5	11322201
11	1103222301
17	110032223301
$i = 5 \bmod 6$	(1,2),(0,(i-5)/6),(3,1),(2,3),(3,(i-5)/6),(0,1),(1,1)

So, we have for the three studied items :

$$L = 6 + \text{int}((i+2)/3) \approx i/3 \quad (1)$$

$$C = \text{if}(i = 3 \bmod 6, 1 + \text{int}((i+5)/6), 3 + \text{int}((i+2)/6)) \approx i/6 \quad (2)$$

$$M = \text{if}(i = 1 \bmod 3, \text{int}((i+2)/3), \text{if}(i = 3 \bmod 6, 1, 2)) \approx \text{or}(i/3, \approx 0) \quad (3)$$

The remarkable point, apart from the entire predictability, is the round trip between a totally distorted palindrome and near-perfect palindrome, regardless of the size of the current number.

If instead of starting from the "exceptional" number 221112, we start from the "ordinary" number 211112, we have the quite different :

$$L \approx 0,47.i \quad (4)$$

$$C \approx 0,23.i \quad (5)$$

$$M \approx 0,11.i \quad (6)$$

Case 2 :

Base 2.

It seems that this base is mainly home to exceptional numbers, namely, a reproducible phase usually begins after a finite number of steps. This phase responds to the following formulas :

$$L = v1 + \text{int}((i+v2)/2) \approx i/2 \quad (7)$$

$$C = \text{int}((i+v3)/4) + \text{if}(i = 0 \bmod 4, v4, \text{if}(i = 1 \bmod 4, v5, \text{if}(i = 2 \bmod 4, v6, v7))) \approx i/4 \quad (8)$$

$$M = \text{int}((i+v8)/4) + \text{if}(i = 0 \bmod 4, v9, \text{if}(i = 1 \bmod 4, v10, \text{if}(i = 2 \bmod 4, v11, v12))) \approx i/4 \quad (9)$$

For example, the (fixed) settings are ($i = 0$ for the initial number), with the reproducible phase starting at the i value given in the "Reprod" column :

Initial number	Reprod.	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	v11	v12
10	$i \geq 20$	6	0	12	0	0	0	-1	4	0	-2	0	0
100	$i \geq 8$	5	0	12	-1	-2	0	0	4	-1	-1	0	-2
1000	$i \geq 18$	7	0	12	0	-1	1	1	4	0	0	1	-1
10000	$i \geq 171$	32	0	64	1	3	-1	-4	20	0	-6	-2	-2
100000	$i \geq 18$	9	0	12	1	0	2	2	4	1	1	2	0
1000000	$i \geq 12$	10	0	12	0	3	1	0	4	2	-1	1	2
10000000	$i \geq 12$	11	0	12	2	1	3	3	4	2	2	3	1
100000000	$i \geq 38$	13	0	12	3	2	4	4	4	3	3	4	2
10000000000	$i \geq 22$	15	0	12	4	3	5	5	4	4	4	5	3
1000000000000	$i \geq 60$	20	0	36	0	-2	2	3	20	0	0	1	-1
10000000000000	$i \geq 22$	17	0	32	0	-1	1	1	12	0	0	1	-3
100000000000000	$i \geq 22$	18	0	36	-1	-3	1	2	20	-1	-1	0	-2
1000000000000000	$i \geq 22$	19	0	32	1	0	2	2	12	1	1	2	-2
10000000000000000	$i \geq 22$	20	0	36	0	-2	2	3	20	0	0	1	-1
100000000000000000	$i \geq 22$	21	0	32	2	1	3	3	12	2	2	3	-1
1000000000000000000	$i \geq 24$	22	0	36	1	-1	3	4	20	1	1	2	0
10000000000000000000	$i \geq 26$	23	0	32	3	2	4	4	12	3	3	4	0
100000000000000000000	$i \geq 28$	24	0	36	2	0	4	5	20	2	2	3	1

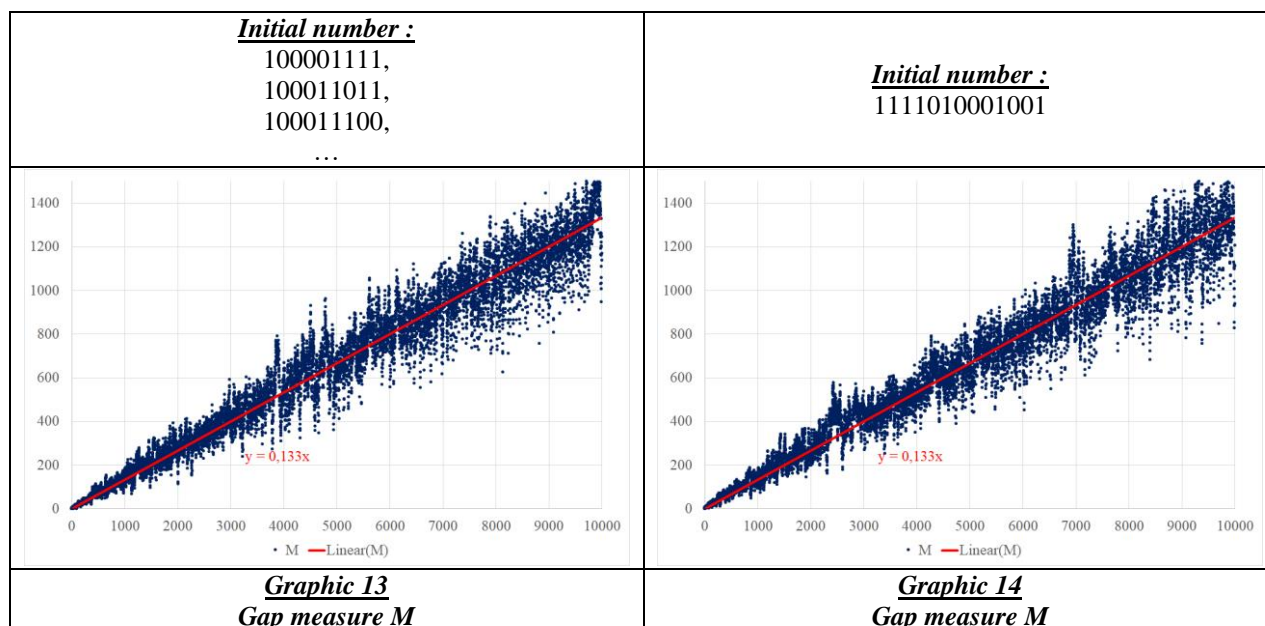
Integers such as the reproducible phase manifests from the beginning ($i \geq 1$) are in order of appearance :

10000111, 10100011, 11000101, 11100001, 100010111, 100101100, 101010011, 101101000, 110010101, 111010001, 1000000111, 1000001111, 1001000111, 1001011100, 1001101100, 1010000011, 1010001011, 1011000011, 1011011000, 1011101000, 1100000101, 1100001101, 1101000101, 1110000001, 1110001001, 1111000001.

This is the complete list of numbers of size 10 or less. These numbers converge on only two threads. Of the 26, the following 4 converge on the second thread :

1000000111, 1010000011, 1100000101, 1110000001.

Among the ordinary numbers, more difficult to find (but not necessarily less dense in N), we have a priori (that is to say for sure as far as $i = 10000$) the following numbers :



In the case of chart 13, the three initial numbers given here all evolve within the same thread. The second graph is a priori another thread (it is at least until step 10000). The growth parameters, on the other hand, are asymptotically identical.

$$L \approx 0,57.i \quad (10)$$

$$C \approx 0,28.i \quad (11)$$

$$M \approx 0,13.i \quad (12)$$

The standard deviations to the average middle line are much larger here than in the case of base 10.

Of the first 1000 integers in base 2 (1 to 11110101000), there are only 41 "normal" numbers all listed below :

100001111	101001100	110100101	1000101111	1010111000	1101010101
100011011	101011000	110110001	1000111100	1011001100	1101100101
100011100	101100011	111001001	1001010111	1011010011	1110011001
100100111	101100100	111100001	1001100111	1011100011	1110101001
100110011	101110000	1000000000	1001110100	1011110000	1111010001
100110100	110001101	1000000001	1010011011	1100011101	1111100001
101001011	110011001	1000011111	1010101011	1100101101	

All of these numbers converge on the same thread.

7.Conclusion.

It seems to us that the question of the existence of Lychrel numbers is a relatively minor point in the study of the addition of a number with its mirror number.

Indeed, we can propose two standard questions, stemming from the end of this study, that replicate and whose character seems to us deeper and more fundamental :

- What is the density of Cheryl's integers (or exceptional numbers) in N in base 2?
- Are all integers Cheryl's integers in base 2?
- What is the density of Cheryl's integers in N in base 4?
- Are all integers Cheryl's integers in base 4?
- What is the density of Cheryl's integers in N in base 10?
- Are all integers Cheryl's integers in base 10?
- What is the density of Cheryl's integers in N in base X ?
- Are all integers Cheryl's integers in base X ?

Indeed, the large number of exceptional type integers in base 2 may suggest that during its growth, an ordinary number is likely to meet an exceptional number and thus changes its growth regime up to infinity. This would then mean the total absence of an ordinary number in this base. If this is indeed the case, this property is likely to be the rule in all bases, even if the accessible data seems to say so far quite the opposite !

References

- [1] Jason Doucette Database <http://jasondoucette.com/worldrecords.html>
- [2] <https://hubertschaetzel.wixsite.com/website>. Page Wade VanLandingham

Appendix 1 : VBA programming.

Programming for base 4 to 10

Public lgw, iter0, v(5000), w(5000), LowBase

Sub Count1()

Base = 10 'base max = 10, base min = 4

Wrdi = "456987689111798120"

LowBase = Base - 1

NumberIteration = 3000

LenWord = Int(NumberIteration / 2) + 100

For i = 0 To NumberIteration: v(i) = 0: w(i) = 0: Next i

Range("B1") = "Base" & Base

Range("B2") = "Len"

Range("C2") = "#1"

Range("D2") = "M"

tot = 0

For i = 0 To NumberIteration

lgw = Len(Wrdi)

Wrdn = StrReverse(Wrdi)

r = 0

Wrd1 = ""

WrdInt1 = 0

For j = lgw To 1 Step -1

WrdInt1 = Val(Mid(Wrdi, j, 1)) + Val(Mid(Wrdn, j, 1)) + r

If WrdInt1 >= Base Then WrdInt1 = WrdInt1 - Base: r = 1: tcarry = tcarry + 1 Else r = 0

Wrd1 = WrdInt1 & Wrd1

Next j

If r = 1 Then Wrds = "1" & Wrd1: r = 0 Else Wrds = Wrd1

For j = 1 To lgw

v(j) = Val(Mid(Wrdi, j, 1))

w(j) = Val(Mid(Wrdi, j, 1))

Next j

k = Int(lgw / 2)

jj = 1

For j = 1 To k

If Abs(w(j) - w(lgw - j + 1)) > 1 Then If Abs(w(j) - w(lgw - j + 1)) <> LowBase Then jj = 2: GoTo Suit01

If Abs(w(j + 1) - w(lgw - j + 1)) > 1 Then If Abs(w(j + 1) - w(lgw - j + 1)) <> LowBase Then jj = 1: GoTo Suit01

Next j

Suit01:

k = Int((lgw + 1 + jj) / 2)

Suit02:

For j = k To lgw Step 1

esp = w(j) - w(lgw - j + jj)

If esp = LowBase Then iter0 = lgw - j + jj: w(iter0) = LowBase: Routine: GoTo Suit02

Next j

Suit04:

For j = lgw To k Step -1

esp = w(lgw - j + jj) - w(j)

If esp = -LowBase Then iter0 = lgw - j + jj: w(iter0) = LowBase: Routine: GoTo Suit04

Next j

For j = lgw To k Step -1

esp = w(lgw - j + jj) - w(j)

If esp = 1 Then w(lgw - j + jj) = w(lgw - j + jj) - 1: GoTo Suit03

If esp = -1 Then w(j) = w(j) - 1

Suit03:

Next j

```

Palind = ""
If jj = 2 Then w(1) = 0
m = 0
r = 0
For j = lgw To 1 Step -1
Palind = Palind & w(lgw - j + 1)
tot = v(j) - w(j) - r
If tot = -LowBase Then r = 1: m = m + 1: GoTo Suit3
If tot = -Base Then r = 1: GoTo Suit3
If tot = 1 Then r = 0: m = m + 1: GoTo Suit3
If tot = 0 Then r = 0: GoTo Suit3
If tot < 0 Then If tot > -LowBase Then r = 1: GoTo Suit3
If tot > 1 Then r = 0
Suit3:
Next j

```

```

Range("H3").Offset(i, 0) = Palind
Range("B3").Offset(i, 2) = m
Range("B3").Offset(i + 1, -1) = "" & Wrds
Range("B3").Offset(i + 1, 0) = Len(Wrds)
Range("B3").Offset(i + 1, 1) = tcarry: tcarry = 0
Wrds = Wrds
Next i
End Sub

```

```

Sub Routine()
For mm = 1 To lgw
w(iter0 - mm) = w(iter0 - mm) - 1
If w(iter0 - mm) = -1 Then w(iter0 - mm) = LowBase Else GoTo Fin
Next mm
Fin:
End Sub

```

Programming for base 2

```

Sub Count2()
Wrds = "11"
tot = 0
For i = 0 To 100
Wrds = StrReverse(Wrds)
r = 0
Wrds1 = ""
For j = Len(Wrds) To 1 Step -1
WrdsInt = Val(Mid(Wrds, j, 1)) + Val(Mid(Wrds, j, 1)) + r
If WrdsInt > 1 Then WrdsInt = WrdsInt - 2: r = 1: tot = tot + 1 Else r = 0
Wrds1 = WrdsInt & Wrds1
Next j
If r = 1 Then Wrds = "1" & Wrds1: r = 0 Else Wrds = Wrds1
m1 = 0: k = Int((Len(Wrds) + 1) / 2): lgw = Len(Wrds)
For j = k To 1 Step -1
If Abs(Val(Mid(Wrds, j, 1)) - Val(Mid(Wrds, lgw - j + 1, 1))) = 1 Then m1 = m1 + 1
Next j
m2 = 1: k = Int((Len(Wrds)) / 2): lgw = Len(Wrds)
For j = k To 1 Step -1
If Abs(Val(Mid(Wrds, j + 1, 1)) - Val(Mid(Wrds, lgw - j + 1, 1))) = 1 Then m2 = m2 + 1
Next j
Range("B3").Offset(i + 1, -1) = "" & Wrds
Range("B3").Offset(i + 1, 0) = Len(Wrds)
Range("B3").Offset(i + 1, 1) = tot: tot = 0
If m2 < m1 Then Range("B3").Offset(i, 2) = m2 Else Range("B3").Offset(i, 2) = m1
Wrds = Wrds

```

```
Next i
End Sub
```

Programming for base 3

```
Public lgw, iter0, v(5000), w(5000), t(5000), LowBase, ww
```

```
Sub Count3()
Base = 3
Wrds = "1"
LowBase = Base - 1
NumberIteration = 3000
LenWord = Int(NumberIteration / 2) + 100
For i = 0 To NumberIteration: v(i) = 0: w(i) = 0: Next i
Range("B1") = "Base" & Base
Range("B2") = "Len"
Range("C2") = "#1"
Range("D2") = "M"
tot = 0
For i = 0 To NumberIteration
lgw = Len(Wrds)
Wrds = StrReverse(Wrds)
r = 0
Wrds = ""
WrdsInt1 = 0
For j = lgw To 1 Step -1
WrdsInt1 = Val(Mid(Wrds, j, 1)) + Val(Mid(Wrds, j, 1)) + r
If WrdsInt1 >= Base Then WrdsInt1 = WrdsInt1 - Base: r = 1: tcarry = tcarry + 1 Else r = 0
Wrds = WrdsInt1 & Wrds
Next j
If r = 1 Then Wrds = "1" & Wrds: r = 0 Else Wrds = Wrds

For j = 1 To lgw
v(j) = Val(Mid(Wrds, j, 1))
w(j) = Val(Mid(Wrds, j, 1))
Next j

jj = 1
k = Int((lgw + 1 + jj) / 2)
Suit02:
'MsgBox ("e")
For j = k To lgw
'MsgBox (j & " " & lgw)
esp = w(j) - w(lgw - j + jj)
If esp = LowBase Then iter0 = lgw - j + jj: w(iter0) = LowBase: Routine: If ww = 1 Then ww = 0: m1 = 1E+100: GoTo
Suit04 Else GoTo Suit02
Next j

Suit04:
For j = k To lgw
esp = w(j) - w(lgw - j + jj)
If esp = -LowBase Then iter0 = j: w(iter0) = LowBase: Routine: If ww = 1 Then ww = 0: m1 = 1E+100: GoTo Suit05
Else GoTo Suit04
Next j

Suit05:
For j = lgw To k Step -1
esp = w(lgw - j + jj) - w(j)
If esp = 1 Then w(lgw - j + jj) = w(lgw - j + jj) - 1: GoTo Suit03
If esp = -1 Then w(j) = w(j) - 1
Suit03:
Next j
```

```

Palind1 = ""
m1 = 0
r = 0
For j = lgw To 1 Step -1
Palind1 = Palind1 & w(lgw - j + 1)
tot = v(j) - w(j) - r
If tot = -LowBase Then r = 1: m1 = m1 + 1: GoTo Suit31
If tot = -Base Then r = 1: GoTo Suit31
If tot = 1 Then r = 0: m1 = m1 + 1: GoTo Suit31
If tot = 0 Then r = 0: GoTo Suit31
If tot = -1 Then r = 1: GoTo Suit31
If tot = -1 Then r = 1: GoTo Suit31
If tot > 1 Then r = 0
Suit31:
Next j

For j = 1 To lgw
t(j) = Val(Mid(Wrdi, j, 1))
Next j

jj = 2
k = Int((lgw + 1 + jj) / 2)
Suit12:
For j = k To lgw
esp = t(j) - t(lgw - j + jj)
If esp = LowBase Then iter0 = lgw - j + jj: t(iter0) = LowBase: Routine: If ww = 1 Then ww = 0: m2 = 1E+100: GoTo
Suit14 Else GoTo Suit12
Next j

Suit14:
For j = k To lgw
esp = t(j) - t(lgw - j + jj)
If esp = -LowBase Then iter0 = j: t(iter0) = LowBase: Routine: If ww = 1 Then ww = 0: m2 = 1E+100: GoTo Suit15 Else
GoTo Suit14
Next j

Suit15:
For j = lgw To k Step -1
esp = t(lgw - j + jj) - t(j)
If esp = 1 Then t(lgw - j + jj) = t(lgw - j + jj) - 1: GoTo Suit13
If esp = -1 Then t(j) = t(j) - 1
Suit13:
Next j

Palind2 = ""
t(1) = 0
m2 = 0
r = 0
For j = lgw To 1 Step -1
Palind2 = Palind2 & t(lgw - j + 1)
tot = v(j) - t(j) - r
If tot = -LowBase Then r = 1: m2 = m2 + 1: GoTo Suit32
If tot = -Base Then r = 1: GoTo Suit32
If tot = 1 Then r = 0: m2 = m2 + 1: GoTo Suit32
If tot = 0 Then r = 0: GoTo Suit32
If tot = -1 Then r = 1: GoTo Suit32
If tot = -1 Then r = 1: GoTo Suit32
If tot > 1 Then r = 0
Suit32:
Next j

If m1 < m2 Then m = m1: Palind = Palind1 Else m = m2: Palind = Palind2

```

```

Range("H3").Offset(i, 0) = Palind
Range("B3").Offset(i, 2) = m
Range("B3").Offset(i + 1, -1) = "" & Wrds
Range("B3").Offset(i + 1, 0) = Len(Wrds)
Range("B3").Offset(i + 1, 1) = tcarry: tcarry = 0
Wrds = Wrds
Next i
End Sub

Sub Routine()
ww = 0
For mm = 1 To lgw
If iter0 - mm < 0 Then ww = 1: GoTo Fin
w(iter0 - mm) = w(iter0 - mm) - 1
If w(iter0 - mm) = -1 Then w(iter0 - mm) = LowBase Else GoTo Fin
Next mm
Fin:
End Sub

```

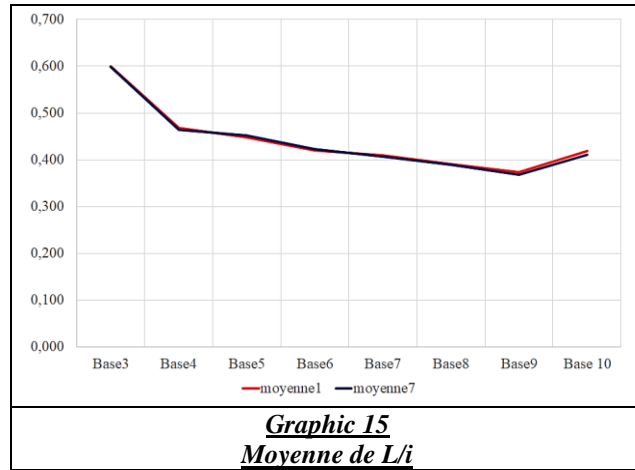
Appendix 2 : Speed of growth in the size of an integer.

We evaluate the L/i ratio of the size L of an integer, i.e. the number of its digits, to the current iteration number i of the Wade VanLandingham algorithm. We assume a single asymptotic L/i growth rate for a given base. The data collected are thus those starting with integer “1” systematically and are presented below.

The k range, on which the k -average is made, is that of the 1000 iterations between the iteration $i = 1000.k$ and the iteration $i = 1000.(k+1)-1$ included. The same goes for the assessment of minimums and maximums.

	mean1	mean2	mean3	mean4	mean5	mean6	mean7
Base 2	0,504	0,502	0,502	0,501	0,501	0,501	0,501
Base 3	0,600	0,599	0,599	0,598	0,598	0,598	0,598
Base 4	0,468	0,469	0,468	0,466	0,463	0,461	0,464
Base 5	0,448	0,446	0,447	0,448	0,449	0,450	0,452
Base 6	0,420	0,421	0,421	0,420	0,421	0,422	0,423
Base 7	0,409	0,407	0,407	0,406	0,407	0,406	0,406
Base 8	0,391	0,392	0,391	0,391	0,390	0,389	0,389
Base 9	0,374	0,371	0,370	0,371	0,370	0,369	0,369
Base 10	0,419	0,418	0,410	0,410	0,412	0,412	0,411
	min1	min2	min3	min4	min5	min6	min7
Base 2	0,503	0,502	0,501	0,501	0,501	0,501	0,501
Base 3	0,599	0,598	0,598	0,598	0,598	0,598	0,598
Base 4	0,464	0,467	0,465	0,463	0,461	0,460	0,462
Base 5	0,445	0,445	0,445	0,447	0,448	0,449	0,451
Base 6	0,417	0,418	0,420	0,420	0,420	0,421	0,422
Base 7	0,406	0,405	0,406	0,405	0,406	0,405	0,405
Base 8	0,388	0,391	0,390	0,390	0,389	0,389	0,389
Base 9	0,370	0,370	0,369	0,370	0,369	0,368	0,368
Base 10	0,415	0,414	0,408	0,409	0,411	0,411	0,410
	max1	max2	max3	max4	max5	max6	max7
Base 2	0,505	0,503	0,502	0,501	0,501	0,501	0,501
Base 3	0,601	0,600	0,599	0,599	0,598	0,598	0,598
Base 4	0,473	0,471	0,470	0,469	0,465	0,463	0,465
Base 5	0,453	0,447	0,448	0,449	0,450	0,451	0,452
Base 6	0,422	0,423	0,422	0,421	0,422	0,422	0,423
Base 7	0,411	0,408	0,408	0,407	0,407	0,407	0,407
Base 8	0,394	0,394	0,393	0,392	0,392	0,390	0,390
Base 9	0,379	0,372	0,371	0,371	0,371	0,370	0,369
Base 10	0,421	0,422	0,414	0,411	0,413	0,413	0,412
	(max1-min1)/min1	(max2-min2)/min2	(max3-min3)/min3	(max4-min4)/min4	(max5-min5)/min5	(max6-min6)/min6	(max7-min7)/min7
Base 2	0,60%	0,22%	0,12%	0,07%	0,05%	0,04%	0,03%
Base 3	0,44%	0,28%	0,23%	0,15%	0,10%	0,10%	0,07%
Base 4	1,77%	0,96%	1,06%	1,23%	0,93%	0,59%	0,51%
Base 5	1,61%	0,48%	0,59%	0,40%	0,51%	0,36%	0,28%
Base 6	1,15%	1,13%	0,55%	0,24%	0,38%	0,18%	0,36%
Base 7	1,26%	0,89%	0,49%	0,43%	0,31%	0,43%	0,40%
Base 8	1,68%	0,82%	0,75%	0,48%	0,76%	0,24%	0,25%
Base 9	2,41%	0,72%	0,31%	0,43%	0,38%	0,44%	0,22%
Base 10	1,57%	1,85%	1,58%	0,53%	0,50%	0,54%	0,50%

Note here that 1 is an exceptional number in base 2. It is therefore not relevant to compare the data corresponding to the other evaluations. Therefore, they are not included in the graph below.



The base comparison is interesting if its understanding makes it possible to find the exact asymptotic value of the L/i ratio. However, the data only reinforce here confusion as a result of the mysterious reversal of the ratio between base 9 and 10.

Another approach is to examine the growth mechanism in more detail. This is largely related to the value of the extreme digits of the series of integers obtained by symmetrical addition.

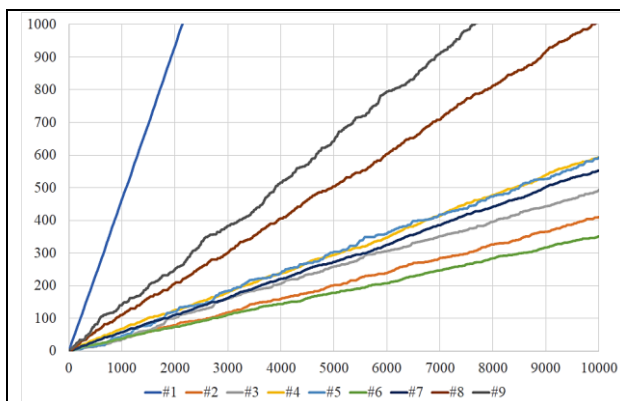
So, in base 10, it is interesting to examine the distribution of digits 0, 1, 2, ... 9 to the right and 1, 2, ..., 9 to the left of the current integer. Several tests based on different initial numbers (here 193 to 197) show that these distributions do not (presumably) depend on the said initial number (for "normal" numbers), what is expected.

Left side	0	1	2	3	4	5	6	7	8	9
193	/	46,87%	4,10%	4,93%	5,92%	5,93%	3,51%	5,52%	10,09%	13,13%
194	/	46,29%	4,07%	5,11%	6,32%	5,68%	4,09%	5,72%	10,40%	12,32%
195	/	46,53%	4,10%	5,01%	6,22%	5,84%	3,89%	5,60%	10,25%	12,56%
196	/	46,51%	3,89%	4,76%	6,28%	5,34%	3,80%	5,79%	10,80%	12,83%
197	/	47,09%	4,33%	5,11%	5,76%	6,13%	3,77%	5,32%	10,00%	12,49%
Average to the left	/	46,66%	4,10%	4,98%	6,10%	5,78%	3,81%	5,59%	10,31%	12,67%

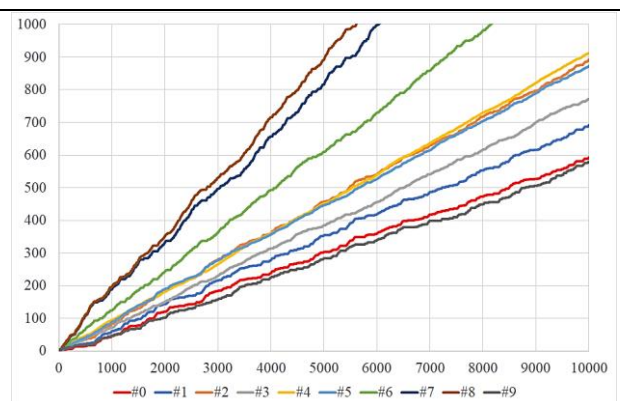
Right side	0	1	2	3	4	5	6	7	8	9
193	5,93%	6,92%	8,92%	7,71%	9,12%	8,73%	12,21%	16,53%	18,14%	5,79%
194	5,68%	6,86%	9,23%	8,06%	9,25%	9,11%	12,71%	16,64%	17,51%	4,95%
195	5,84%	6,92%	9,25%	7,82%	9,22%	9,03%	12,45%	16,78%	17,67%	5,02%
196	5,34%	6,42%	8,75%	8,01%	9,29%	8,82%	13,02%	17,02%	18,23%	5,10%
197	6,13%	7,18%	9,56%	7,61%	9,32%	8,61%	12,27%	16,07%	17,48%	5,77%
Average to the right	5,78%	6,86%	9,14%	7,84%	9,24%	8,86%	12,53%	16,61%	17,81%	5,33%

The evolution of the population of each digit-class (0 to 9) is somewhat linear as shown in the graphs below. This is expected.

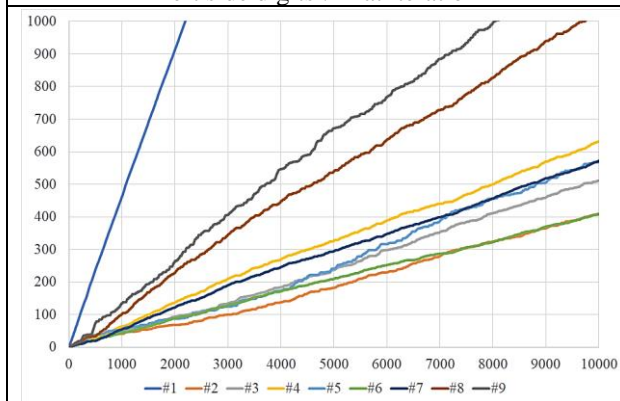
On the other hand, the expected existence, except for 1 on the left which must logically be the majorant in terms of quantity, of an equiprobable distribution between the different digits is not the effective case. Digits 8 and 9 are more common (except 1) on the left, while 7 and 8 are more common on the right.



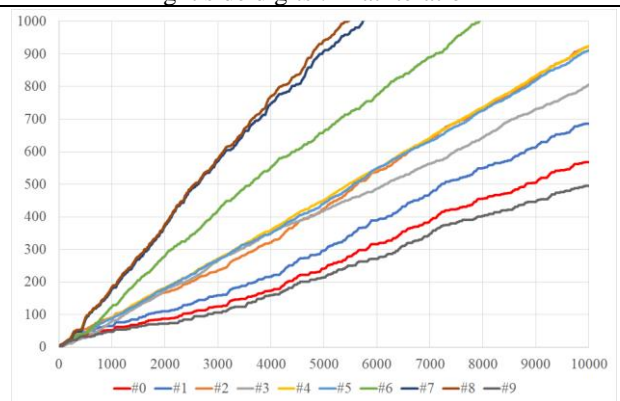
Graphic 16
Initial number : 193
10000 iterations
Left side digits : #i at iteration i



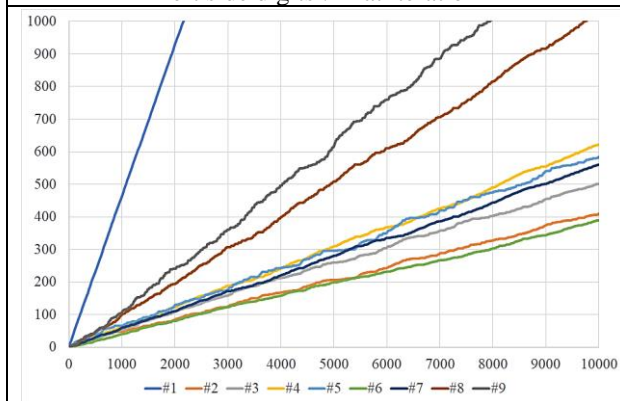
Graphic 17
Initial number : 193
10000 iterations
Right side digits : #i at iteration i



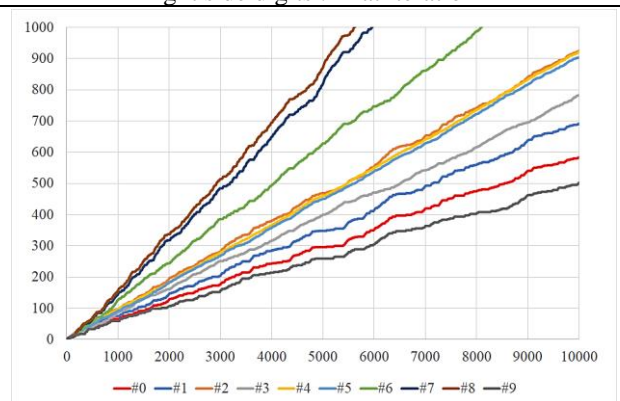
Graphic 18
Initial number : 194
10000 iterations
Left side digits : #i at iteration i



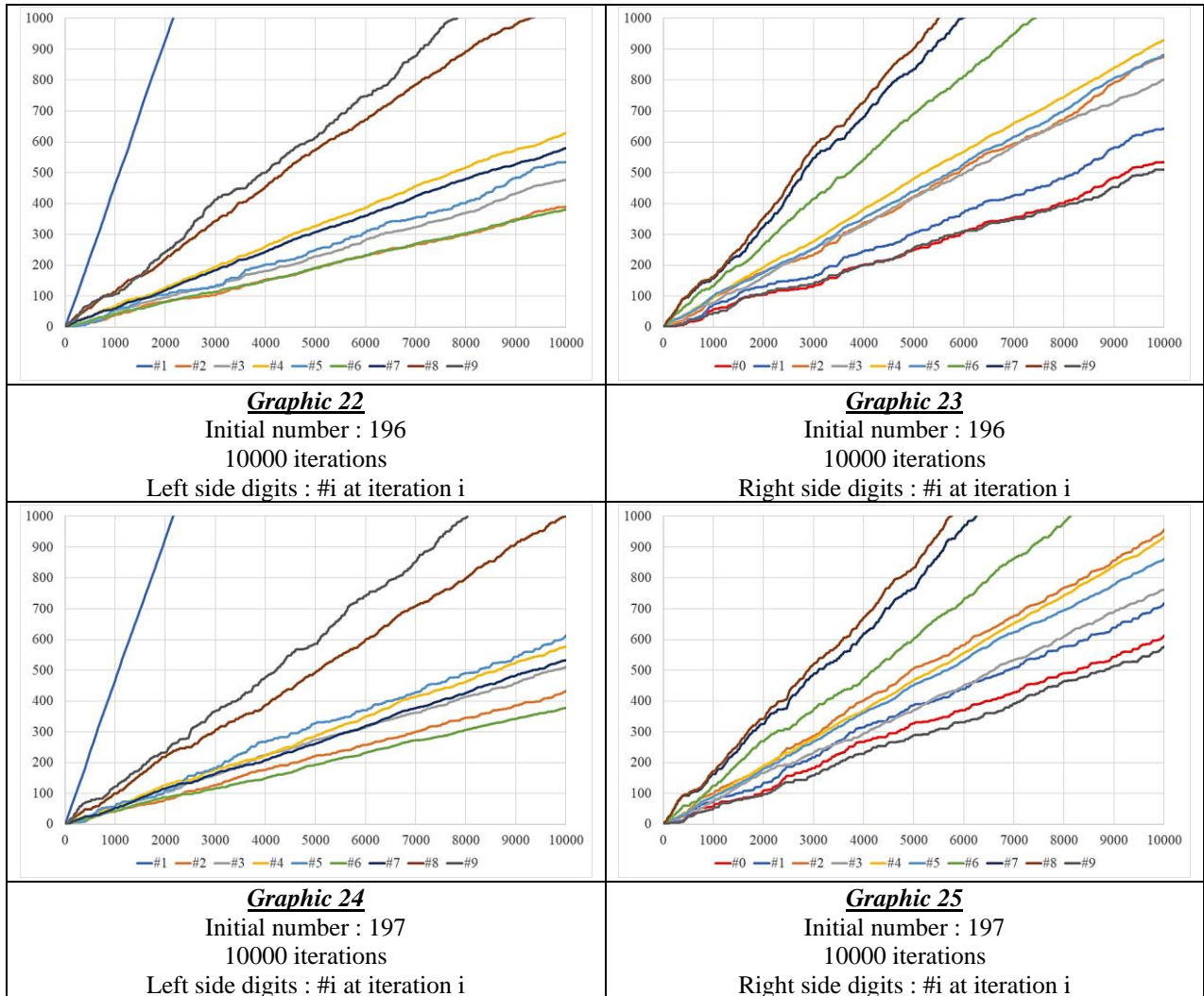
Graphic 19
Initial number : 194
10000 iterations
Right side digits : #i at iteration i



Graphic 20
Initial number : 195
10000 iterations
Left side digits : #i at iteration i



Graphic 21
Initial number : 195
10000 iterations
Right side digits : #i at iteration i



By crossing the percentages of the distributions of the digits 1 to 9 on the left with those 0 to 9 on the right, an approximate value of the length growth factor L is obtained by summing up the percentages for which the sum of the two digits on the left and right is greater than or equal to 10. The table below highlights the terms to add :

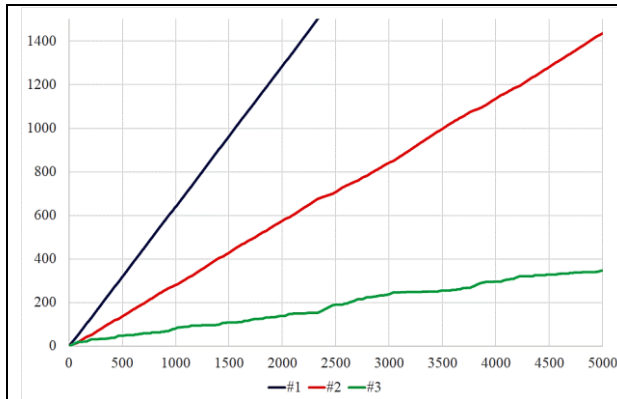
	Left digits	1	2	3	4	5	6	7	8	9
Right digits	Left mean Right mean	46,66%	4,10%	4,98%	6,10%	5,78%	3,81%	5,59%	10,31%	12,67%
0	5,78%	2,70%	0,24%	0,29%	0,35%	0,33%	0,22%	0,32%	0,60%	0,73%
1	6,86%	3,20%	0,28%	0,34%	0,42%	0,40%	0,26%	0,38%	0,71%	0,87%
2	9,14%	4,27%	0,37%	0,46%	0,56%	0,53%	0,35%	0,51%	0,94%	1,16%
3	7,84%	3,66%	0,32%	0,39%	0,48%	0,45%	0,30%	0,44%	0,81%	0,99%
4	9,24%	4,31%	0,38%	0,46%	0,56%	0,53%	0,35%	0,52%	0,95%	1,17%
5	8,86%	4,13%	0,36%	0,44%	0,54%	0,51%	0,34%	0,50%	0,91%	1,12%
6	12,53%	5,85%	0,51%	0,62%	0,76%	0,72%	0,48%	0,70%	1,29%	1,59%
7	16,61%	7,75%	0,68%	0,83%	1,01%	0,96%	0,63%	0,93%	1,71%	2,10%
8	17,81%	8,31%	0,73%	0,89%	1,09%	1,03%	0,68%	1,00%	1,84%	2,26%
9	5,33%	2,49%	0,22%	0,27%	0,32%	0,31%	0,20%	0,30%	0,55%	0,67%

The double-framed total is 100% while the highlighted percentages add up to 40.13%, thus a ratio of 0.401 to be compared to 0.413 the average of the values obtained above (page 16). The difference is due to the contribution of second ranked carries. This contribution is in the order of magnitude of 1%.

When the base is smaller, the alignment of values is more delicate.
For example, in base 4, we have the following distributions :

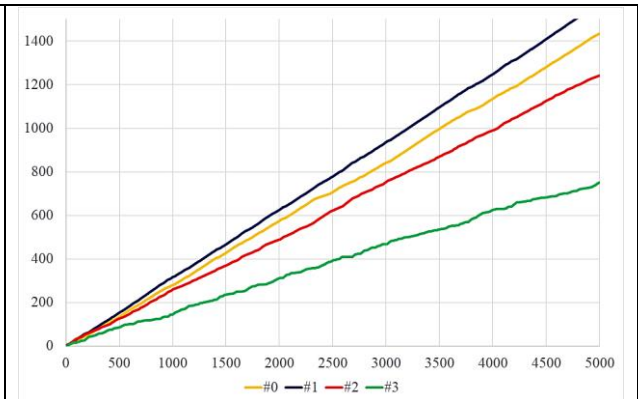
Left side	0	1	2	3
#1	/	64,4%	28,7%	6,92%

Right side	0	1	2	3
#1	28,7%	31,4%	24,8%	15,0%



Graphic 26

Initial number : 1
5000 iterations
Left side digits : #i at iteration i



Graphic 27

Initial number : 1
5000 iterations
Right side digits : #i at iteration i

The crossboard gives :

Left digits		1	2	3
Right digits	Left mean	64,4%	28,7%	6,92%
	Right mean	28,7%	31,4%	24,8%
0	28,7%	18,5%	8,2%	2,0%
1	31,4%	20,2%	9,0%	2,2%
2	24,8%	16,0%	7,1%	1,7%
3	15,0%	9,7%	4,3%	1,0%

The double-framed total is 100% while the highlighted percentages add up now to 26.1%, thus a ratio of 0,261 to be compared to 0.465, the average of the values obtained above (page 16). The difference can be explained by the contribution of the second ranked carries, which is absolutely no longer negligible here.